MULTIOBJECTIVE OPTIMIZATION: HISTORY AND PROMISE

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Abstract

Francis Y. Edgeworth (1845-1926) and Vilfredo Pareto (1848-1923) are credited for first introducing the concept of non-inferiority in the context of economics. Since then multiobjective optimization has permeated engineering and design and has developed at a rapidly increasing pace. This paper gives a brief review of the history of multiobjective optimization and motivates its importance in the context of the engineering and design of complex systems. A brief review of methods distinguishes between Scalarization and Pareto approaches. These are primarily distinguished by ways in which designer (or customer) preferences are brought into the design optimization process. This review is not intended to be comprehensive, but focuses on the most popular multiobjective methods. The Karush-Kuhn-Tucker (KKT) optimality conditions for multiple objectives and the associated role of Lagrange multipliers and preference weights are briefly illuminated. Two emerging trends in multiobjective optimization are presented: the inclusion of manufacturing cost alongside performance considerations as well as the emergence of adaptive multiobjective algorithms. Applications of interest to the author include structural design, communications satellites, radio telescope arrays and automotive platforms. The importance of Pareto frontiers goes far beyond finding a "best design" or "set of non-dominated" solutions and includes analysis of technology infusion in existing systems, system architecture selection as well as lifecycle engineering of systems for reconfigurability, platforming and extensibility.

Keywords: Multiobjective Optimization, Pareto Frontier, Weighted Sum Method, Utility Theory

1. Introduction

A generic multi-objective design optimization problem may be formulated as in Eq. (1):

$$\begin{aligned} &\min \mathbf{J}(\mathbf{x},\mathbf{p}) && \text{where} \quad \mathbf{J} = \left[J_1(\mathbf{x}) \quad \cdots \quad J_z(\mathbf{x}) \right]^T \\ &\text{s.t.} \quad \mathbf{g}(\mathbf{x},\mathbf{p}) \le 0 && \qquad\qquad \mathbf{x} = \left[x_1 \quad \cdots \quad x_i \quad \cdots \quad x_n \right]^T \\ &\qquad \mathbf{h}(\mathbf{x},\mathbf{p}) = 0 && \qquad\qquad \mathbf{g} = \left[g_1(\mathbf{x}) \cdots g_{m_1}(\mathbf{x}) \right]^T \\ &x_{i,LB} \le x_i \le x_{i,UB} \quad (i = 1,\dots,n) && \qquad\qquad \mathbf{h} = \left[h_1(\mathbf{x}) \cdots h_{m_2}(\mathbf{x}) \right]^T \\ &\mathbf{x} \in S && \end{aligned} \tag{1}$$

Here, **J** is a column vector of z objectives, whereby $J_i \in \mathbb{R}$. The individual objectives are dependent on a vector **x** of n design variables as well a vector of fixed parameters, **p**. The individual design variables are assumed continuous and can be changed independently by a designer within upper and lower bounds, $\mathbf{x}_{UB}$ and $\mathbf{x}_{LB}$, respectively. In order for a particular design **x** to be in the feasible domain S, both a vector of m_1 inequality constraints, **g**, and m_2 equality constraints, **h**, have to be satisfied. The problem is to minimize – simultaneously – all elements of the objective vector.

A number of names have been given to this type of problem: vector minimization, multi-criteria optimization, multi-attribute maximization and so forth. For the most part these are synonymous and we will refer collectively to this class of problems as *multiobjective optimization* (MOO) problems.

Why should one care about this class of problems in the first place? The answer – at first – is very obvious. All design and engineering activity is fundamentally multi-objective in nature because of the existence of inherent tensions between the four main objectives in product or system design (Fig.1): performance, cost, schedule and risk (Maier & Rechtin, 2000). With schedule and risk levels (e.g. probability of failure of a component) fixed, better performance can generally only be achieved by increasing cost. Pulling along one of the dimensions in the diagram of Fig.1 generally requires compromises along the other dimensions.

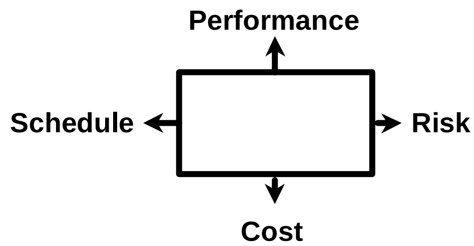

Fig. 1: Tensions during system design

Trading off schedule, cost and product performance is generally considered the domain of project management. Trading of various dimensions of performance with each other as well as with cost and risk is generally considered as the domain of Engineering. Quantifying, visualizing and resolving tradeoffs is one of the key duties of system designers.

An emphasis on multiobjective thinking helps avoid potentially sub-optimal point designs. One may argue that one can circumvent the dilemma posed by a multiobjective problem[1] by selecting the most important objective from **J** and converting the other objectives to constraints. This often requires setting arbitrary constraint levels during early design and such artificial constraints may not truly exist. An example of an existing complex system where multiple – conflicting – objectives had to be met is the F/A-18 aircraft. This system has both continuous design variables (aspect ratio, dihedral angle, engine thrust level and so forth) as well as many discrete design variables (number and location of engines, fuselage splice locations …). These objectives capture both the operational performance of the aircraft (top speed, range, payload capability, stall speed, radar cross section…) as well as its lifecycle objectives (mean-time-between-failure, maintainability, cost-per-flight-hour and avionics growth potential, among others …).

Furthermore, it is important to distinguish between multiobjective and multidisciplinary design situations (Fig.2).

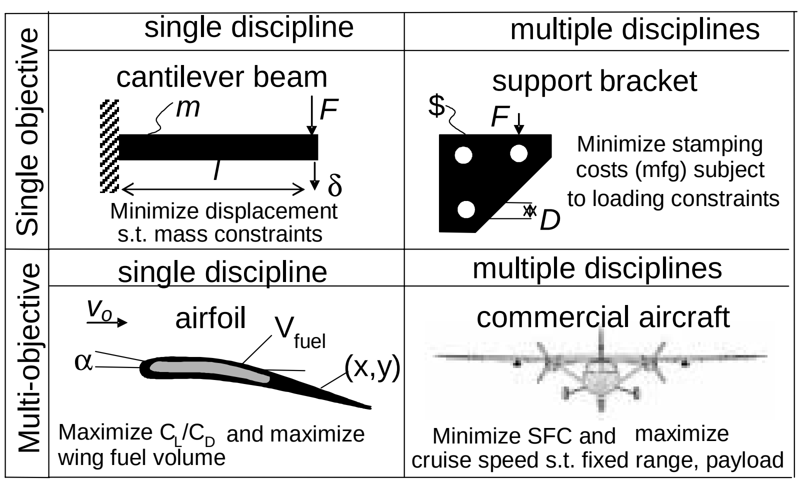

Fig.2: Multiobjective, multidisciplinary situations are prevalent in system-level design situations (lower right), while single discipline, single objective design often applies only at the component level (upper left)

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[1] generally there will not be a single solution, **x***, that is optimal along all dimensions

The following - strongly non-linear - example illustrates some of the difficulties in solving multi-objective optimization problems. We seek to simultaneously maximize two “peaks” functions within the two-dimensional interval [-3, 3]:

$$\begin{aligned} &\max \mathbf{J}(\mathbf{x}) = \begin{bmatrix} J_1 & J_2 \end{bmatrix}^T \\ &\text{s.t.} \quad -3 \le x_i \le 3 \quad i = 1,2 \end{aligned} \tag{2}$$

$$J_1 = 3(1-x_1)^2 e^{-x_1^2-(x_2+1)^2} - 10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right)e^{-x_1^2-x_2^2} - 3e^{-(x_1+2)^2-x_2^2} + 0.5(2x_1 + x_2) \tag{3a}$$

$$J_2 = 3(1+x_2)^2 e^{x_2^2-(x_1+1)^2} - 10\left(-\frac{x_2}{5} + x_2^3 - x_1^5\right)e^{x_2^2-x_1^2} - 3e^{-(2-x_2)^2-x_1^2} \tag{3b}$$

Fig. 3 shows the design space for each objective along with the optimal solutions for each objective, taken alone:

$$\begin{aligned} &\mathbf{x}^{1*} = \begin{bmatrix} 0.0532 & 1.5973 \end{bmatrix}^T \text{ with } \mathbf{J}(\mathbf{x}^{1*}) = \begin{bmatrix} 8.9280 & -4.8202 \end{bmatrix}^T \\ &\mathbf{x}^{2*} = \begin{bmatrix} -1.5808 & 0.0095 \end{bmatrix}^T \text{ with } \mathbf{J}(\mathbf{x}^{2*}) = \begin{bmatrix} -6.4858 & 8.1118 \end{bmatrix}^T \end{aligned} \tag{4}$$

While the first solution maximizes J_1, the second maximizes J_2. The problem is that the value of J_2 at $\mathbf{x}^{1*}$ is very low and the value of J_1 evaluated at $\mathbf{x}^{2*}$ is also low. There exists no single solution $\mathbf{x}^*$ that maximizes both J_1 and J_2 at the same time.

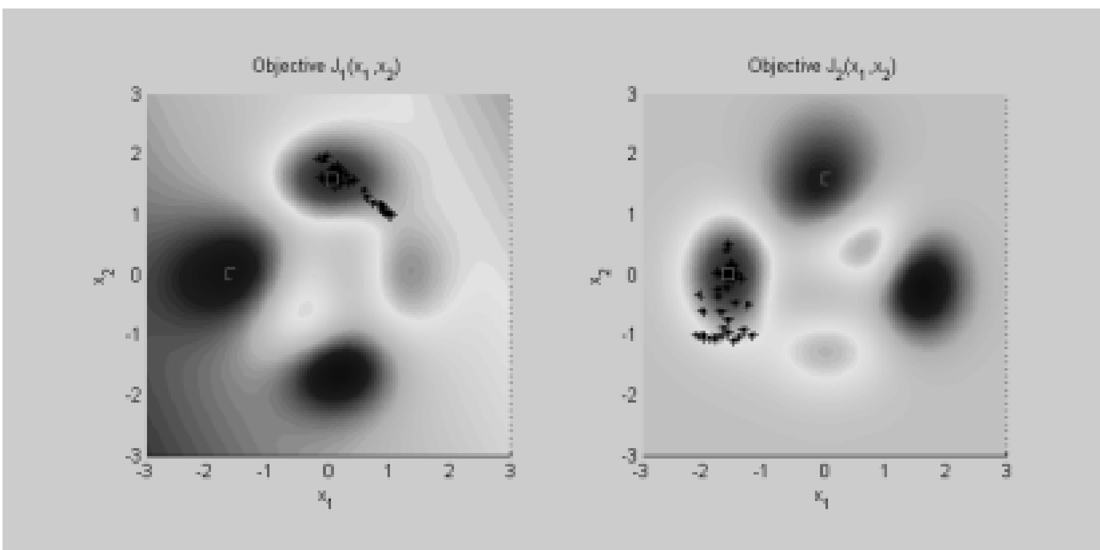


Fig.3: Function J_1 with optimal solution $\mathbf{x}^{1*}$ (left) and function J_2 with optimal solution $\mathbf{x}^{2*}$ (right). Black dots indicate the final population of a genetic algorithm used to identify the maxima of both functions. Squares indicate optima.

One potential solution is to form an aggregate objective function, containing contributions from both J_1 and J_2, and to find its optimum. An objective function $J_{tot}=J_1+J_2$, for example, exhibits an optimum “tradeoff” solution:

$$\mathbf{x}^{\mathbf{tot}*} = \begin{bmatrix} 0.8731 & 0.5664 \end{bmatrix}^T \quad \text{with} \quad J_{tot}(\mathbf{x}^{\mathbf{tot}*}) = 6.1439 \text{ and } \mathbf{J}(\mathbf{x}^{\mathbf{tot}*}) = \begin{bmatrix} 3.0173 & 3.1267 \end{bmatrix}^T \tag{5}$$

The values of each objective are lower than previously, but the aggregate – with equal weighting – is maximized. While this approach is relatively primitive, it is also most common. Other more sophisticated methods have been developed.

2. History

Rational people attempt to make the “best” decision within a specified set of possible alternatives. Historically, “best” has been defined differently in different fields. In economics, where multiobjective thinking arguably originated, the “best” referred to decisions taken by buyers and sellers (micro-economics) or governments (macro-economics), which simultaneously optimize or balance several criteria. Taxation is a good example. An optimal, average level of tax collected (% per $ of economic activity) maximizes the revenue available for the common good, while maintaining a sufficient incentive for individuals to earn income from their own work. One of the first individuals to consider such tradeoffs was F.Y Edgeworth (Fig. 4 left).

Fig.4 (left) Francis Y. Edgeworth (1845-1926) and (right) Vilfredo Pareto (1848-1923)

In 1881 at King’s College (London) and later at Oxford, economics Professor F.Y. Edgeworth was the first to define an optimum for multicriteria economic decision making (Edgeworth 1881). He did so for the multi-utility problem within the context of two hypothetical consumer criteria, P and π: *“It is required to find a point (x,y,) such that in whatever direction we take an infinitely small step, P and π do not increase together but that, while one increases, the other decreases.”* Pareto on the other hand was a contemporary of Edgeworth, born in Paris in 1848 to a French mother and Genovese father. He graduated from the University of Turin in 1870 with a degree in Civil Engineering and a thesis with the title: “The Fundamental Principles of Equilibrium in Solid Bodies”.[2] While working in Florence as a civil engineer from 1870-1893, Pareto took up the study of philosophy and politics and was one of the first to analyze economic problems with mathematical tools. In 1893, Pareto became the Chair of Political Economy at the University of Lausanne in Switzerland, where he created his two most famous theories: *Circulation of the Elites* and *The Pareto Optimum.* While the first remains controversial to this day due to some racial undertones, the second has found broad acceptance (Pareto 1906): “*The optimum allocation of the resources of a society is not attained so long as it is possible to make at least one individual better off in his own estimation while keeping others as well off as before in*

[2] One may speculate that the notion of force and torque equilibrium in static structures may have stimulated Pareto to think about equilibria in the larger econo-political context. The generalized Karush-Kuhn-Tucker optimality conditions (Eq. 13-16) also embody the notion of equilibrium between objective and constraint gradients. The KKT conditions, however, were only formulated after Pareto’s death. The term “Pareto Frontier” is attributed to him.

their own estimation."

The translation of Pareto's work into English in 1971 spurred the development of multi-objective methods in Applied Mathematics and Engineering. The growth of this field manifested itself particularly strongly in the United States with pioneering contributions by (Stadler 1979), (Steuer 1985) and many others. Another hot spot of activity and progress, particularly in the theoretical aspects of multiobjective optimization can be found in Japan (Sawaragi, Nakayama and Tanino, 1985). Over the last three decades the applications of multiobjective optimization have grown steadily in many areas of Engineering and Design. The advent of the internet and a number of focused conferences on the topic have also contributed to the formation of a community of researchers and practitioners in multiobjective optimization. A particularly remarkable resource in this area is the website created and maintained by (Coello-Coello 2004).

3. Scalarization Methods

There is general consensus that multiobjective optimization methods can be broadly decomposed into two categories: Scalarization approaches and Pareto approaches. While different names are used for these categories, the fundamental discriminator is always the same. In the first group of methods the multiobjective problem is *solved by translating it back to a single (or a series of) single objective, scalar problems*. This requires the formation of an overarching objective function which contains contributions from the sub-objectives in vector **J**. The formation of the aggregate objective function requires that the preferences or weights between objectives are assigned apriori, i.e. before the results of the optimization process are known. The Pareto methods, on the other hand, keep the elements of the objective vector **J** separate throughout the optimization process and typically use the concept of dominance to distinguish between inferior and non-inferior solutions. Methods where preferences are expressed during optimization represent a third, albeit less well developed category. The end goal of all these methods is the same: to provide designers and decision makers with a set of `optimal' alternatives to choose from. This recognizes the fact that design alternatives – in practice – are also selected on the basis of objectives which may not be contained in **J**, perhaps because they are not easily quantifiable. Table 1 provides and overview of Multiobjective Optimization Methods.

Table 1: Overview of Multiobjective Methods – underlined methods are mentioned in this paper

Scalarization Methods (apriori preference expression)	**Pareto Methods (a-posteriori preference expression)**
<u>Weighted Sum Approach</u>	<u>Exploration and Pareto Filtering</u>
Compromise Programming (Non-linear combinations)	Weighted Sum Approach (with weight scanning)
<u>Multiattribute Utility Analysis (MAUA) – Utility Theory</u>	<u>Adaptive Weighted Sum method (AWS)</u>
Physical Programming, Goal Programming	<u>Normal Boundary Intersection (NBI)</u>
Lexicographic Approaches	<u>Multiobjective Genetic Algorithms (MOGA)</u>
Acceptability Functions, Fuzzy Logic	Multiobjective Simulated Annealing (MOSA)

A comprehensive overview and comparison of these methods can be found in (Andersson 2001). Here, we briefly discuss a few representatives to highlight some of the prevalent issues associated with the current state-of-the-art.

Scalarization methods are based on the assumptions that (i) designer or decision-maker preferences are known before design solutions are found and that (ii) the z objectives can be meaningfully combined to express a utility, U, a dimensionless scalar quantity expressing the goodness of a particular design (Eq. 6).

$$\begin{aligned} &\max \left\{ U\left(J_1, J_2, \ldots, J_z\right) \right\} \\ &s.t. \quad J_i = f_i\left(\mathbf{x}, \mathbf{p}\right) \quad 1 \le \mathrm{i} \le \mathrm{z} \\ &\qquad \mathbf{x} \in S, \quad U \in \mathbb{R}^+ \end{aligned} \tag{6}$$

The easiest to understand – and most widely used – Scalarization method is the Weighted Sum (WS) approach:

$$\begin{aligned} &\max U\left(\mathbf{J}\left(\mathbf{x}, \mathbf{p}\right)\right) \\ &\text{where } U = \sum_{j=1}^{z} \lambda_j \frac{J_j}{sf_j} \text{ with } \boldsymbol{\lambda} = \left[\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_z\right]^T \\ &\text{and } \boldsymbol{\lambda} \in \mathbb{R}^z \left| \lambda_i > 0, \sum_{i=1}^{z} \lambda_i = 1 \right. \\ &\text{and } \mathbf{x} \in S \end{aligned} \tag{7}$$

Formulated in this way the aggregate objective U always forms a strictly convex combination of objectives. This is achieved by ensuring that all (preference) weights, λ_j, add to unity and are themselves positive scalars. One of the issues in this method is the appropriate choice of scaling factors sf_j for all constituent objectives. In the case of two equally scaled objectives, Eq. (7) simplifies to the well known form:

$$U = \lambda J_1 + \left(1 - \lambda\right) J_2 \tag{8}$$

Finding optima for U as λ is changed gradually, in equal intervals, from 0 → 1 reveals a set of optimal solutions as the weight is gradually shifted from one objective to another. We may apply this method to the numerical example presented in Eq. (2,3) with the resulting objective space shown in Fig. 5.

The series of optima (obtained with a weight increment $\Delta\lambda$=0.05) are identified towards the upper right corner of the graph (maximize both J_1 and J_2) and are connected by straight line segments. The black dots indicate results of a full factorial analysis on the interval [-3,3]. The optimum obtained for λ=0 is $\mathbf{x}^{2*}$ (upper left), while the optimum for λ=1 is $\mathbf{x}^{1*}$ (lower right), see Eq. (4). This illustrates that interesting solutions are found, but unfortunately:

1. Many interesting Pareto points are missed
2. The resulting optima are unevenly distributed

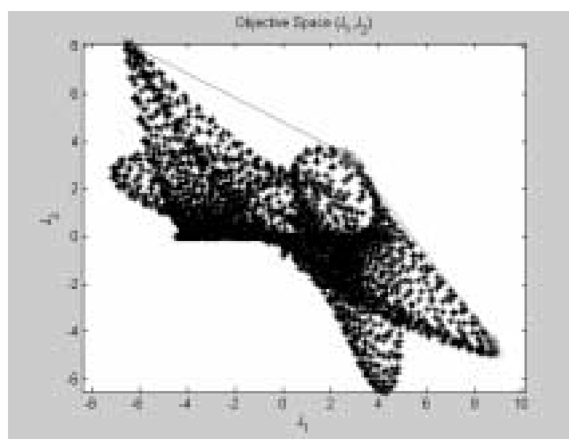
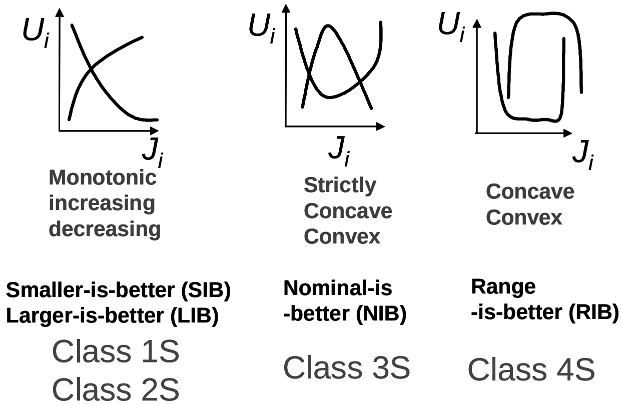


Fig.5: Optimal solutions from Weighted Sum Approach

A more comprehensive approach, underpinned by utility theory, is based on the mathematical construction of a utility function which allows non-linear combinations of objectives via intermediate utility functions. The most prevalent shapes of these utility functions have been classified by various researchers as shown in Fig. 6.



Monotonically increasing or decreasing relationships between an objective J_i and its corresponding utility U_i are captured by larger-is-better or smaller-is-better relationships, while (strictly) convex or concave functions capture a nominal-is-better or in range-is-better type of utility. There also exist non-monotonic utility functions to capture periodic utilities, but these are special cases that are encountered infrequently in practice. Multiattribute utility analysis (MAUA) is one popular method.

Fig.6: Types of utility functions, (Cook 1997, Messac 2000)

In multiattribute utility analysis (MAUA) the total utility of a design solution is a scalar on the interval between 0 (no utility) and 1 (highest utility). This scalar is a weighted sum of partial utilities obtained by mapping the raw objectives J_i to the utility functions shown in Fig. 6. For two objectives we obtain the special case:

$$U\left(J_1, J_2\right) = Kk_1k_2U_1(J_1)U_2(J_2) + k_1U_1(J_1) + k_2U_2(J_2) \tag{9}$$

where k1 and k2 are the individual weights corresponding to J1 and J2, respectively. The scalar K

$$K = (1 - k_1 - k_2) / k_1k_2 \tag{10}$$

is needed to scale the cross-utility term to ensure that the overall utility remains in the interval [0,1]. While utility optimization is effective and widely used it requires extensive interviews to determine appropriate utility functions (Fig.6) and weights (Eq. 9, 10). Once the utility function has been constructed, optimization can occur and the design with maximal utility can be found. One of the dangers of this approach is that decision makers will be influenced by the ways in which the utility interviews are conducted. Also, shifts in preferences can occur once the set of feasible designs becomes known. Most – if not all – Scalarization approaches can be represented via the utility function approach..

4. Pareto Methods

Pareto methods attempt to find a set of efficient solutions, $\mathbf{x}^{*\mathbf{j}}$, such that the objective vectors corresponding to those solutions are non-dominated in *z*-dimensional objective space. Dominance (for maximization) is defined as follows:

Let $\mathbf{J^1}, \mathbf{J^2} \in \mathbb{R}^z$ be two feasible objective vectors. Then $\mathbf{J^1}$ dominates $\mathbf{J^2}$ (weakly) iff

$$\mathbf{J^1} \geq \mathbf{J^2} \quad \text{and} \quad \mathbf{J^1} \neq \mathbf{J^2} \tag{11}$$

or, more precisely:

$$\begin{aligned} &J_i^1 \geq J_i^2 \quad \forall\, i \\ &\text{and } \ J_i^1 > J_i^2 \ \text{ for at least one } i \end{aligned} \tag{12}$$

For strong dominance all elements of $\mathbf{J^1}$ would have to be greater than the corresponding elements of $\mathbf{J^2}$.

Based on the notion of dominance, the simplest approach is a combination of *design space exploration and dominance (Pareto) filtering*. This has been applied to our numerical example (Eq. 2,3) and is shown in Fig. 7.

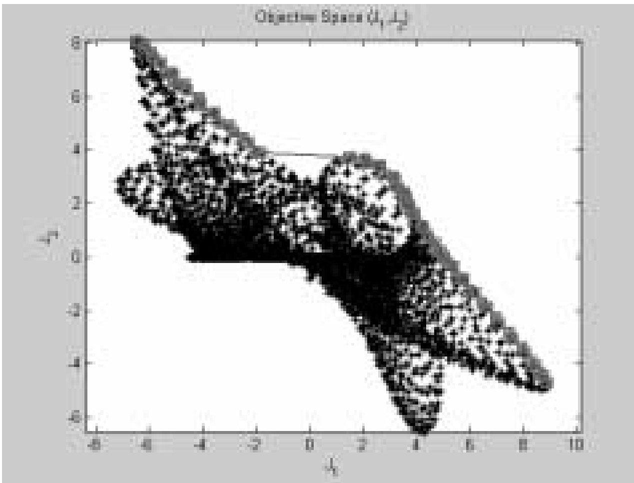


Fig.7: Approximation to Pareto Front via dominance filtering

A comparison with Fig.5 shows a much more complete approximation of the Pareto front of non-dominated solutions. This is appealing, but raises two important points:

1. A comprehensive or full-factorial evaluation of the design space is often impossible due to the *n*-dimensionality of the design vector, **x**, and the required computational effort for obtaining **J, g** and **h**.
2. The solutions obtained in this way are mere approximations of the Pareto Front. More precisely, the points only satisfy non-dominance (Eq. 11, 12).

A stronger condition is to seek solutions that satisfy the multi-objective version of the Karush-Kuhn-Tucker (KKT) optimality conditions:

If **x*** is non-inferior (=Pareto optimal) it satisfies the following KKT conditions:

a.) **x*** is feasible, i.e. $\mathbf{x}^* \in S$ and $S = \varnothing$ (13)

b.) all objective functions J_i and constraints g_j are differentiable[3] (14)

c.) At **x*** the constraints are satisfied $g_j\left(\mathbf{x}^*\right) \leq 0 \;\; \forall \;\; j = 1, 2, ..., m$ (15)

and $\lambda_j g_j\left(\mathbf{x}^*\right) = 0$ whereby $\lambda_j \geq 0 \; \forall \;\; j = 1, .., m$

d.) There exist $\mu_i \geq 0 \; \forall \;\; i = 1, .., n$ with strict inequality holding for at least one *i* such that (16)

the condition $\sum_{i=1}^{n} \mu_i \nabla J_i(x^*) + \sum_{j=1}^{m} \lambda_j \nabla g_j(x^*) = 0$ is true.

The condition described in Eq. (16) expresses the fact that the gradients of the objectives and gradients of the constraints are in equilibrium with each other at a Pareto-optimal point. Note, that among multipliers, the preferences μ_i are the corollary to the weights (λ_i) discussed in Eq. (7), while the λ_j's in Eq. (16) are the Lagrange multipliers.

Among the Pareto approaches shown in Table 1 (right column) two in particular have gained increased acceptance and use in recent years: Normal Boundary Intersection (NBI) as well as Multiobjective Genetic Algorithms (MOGA). While NBI (Das and Dennis 1998) relies on equality constraints normal to a line connecting the anchor points (e.g. $\mathbf{x}^{1*}$ and $\mathbf{x}^{2*}$) in objective space, MOGA's evolve populations of designs gradually so that they approximate a Pareto frontier as closely as possible. NBI and MOGA results for the numerical example, Eq. (2,3), are shown in Fig.8.

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[3] All equality constraints **h** can be transformed into pairs of related inequality constraints **g**.

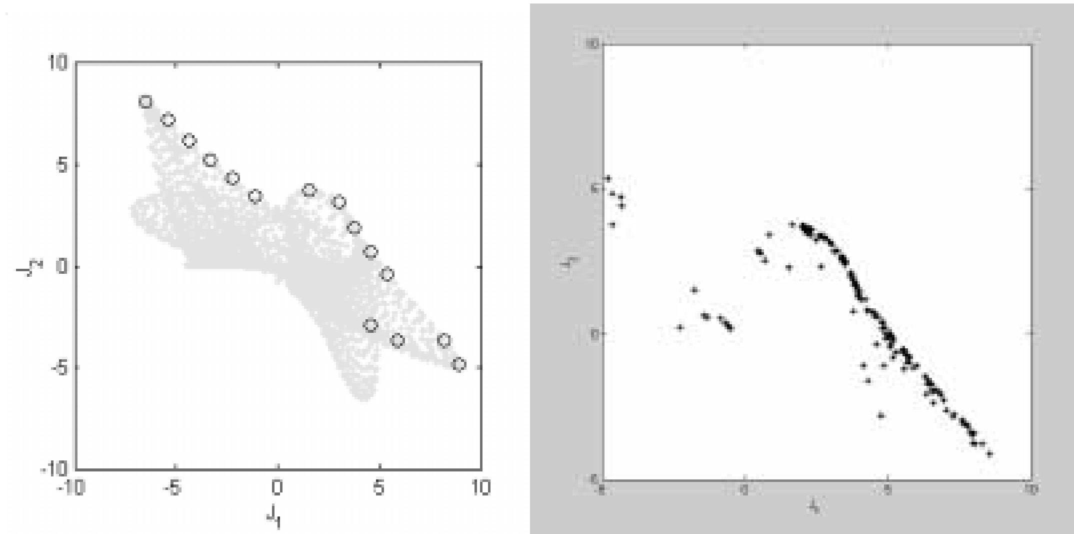

Fig. 8: (left) Optimization results obtained with NBI, (right) results obtained with MOGA (10 generations)

While both of these methods are effective and don't require apriori assignments of weights, there are issues with both methods. In the case of NBI a nearly uniform representation of the Pareto front is usually obtained. However, due to its reliance on equality constraints NBI will converge to local optima for complex, non-linear problems resulting in dominated or non-Pareto points (Fig.8 left). In the case of MOGA the main issues are the large computational expense as well as a tendency for niching (clumping of solutions in objective space) which results in underrepresented regions of the Pareto front (Fig. 8 right). All of these issues (and others) are subjects of ongoing research in the multiobjective optimization community.

5. Emerging Trends: Manufacturing Cost, Adaptivity

Among the new developments in multiobjective optimization we will focus on two in particular: the inclusion of manufacturing cost, as well as the emergence of adaptivity in multiobjective optimization algorithms.

5.1 Manufacturing Cost

Traditionally, structural design and optimization have focused on maximization of structural performance alone (compliance, displacements, natural frequencies/stiffness), subject to a variety of stress and perhaps mass constraints. Increasingly there is recognition that structural components must also be cost effective in terms of manufacturing. One of the trends in multiobjective, structural optimization is the inclusion of manufacturing cost as an objective which is on par with structural performance (Kim, Nadir and de Weck 2004). The tradeoff between manufacturing cost and structural performance for a simple fixed-free cantilever with end load is shown in Fig.9. All three designs have the same mass and are subjected to the same tip load. The tradeoff is between tip displacement (x-axis, J_1) and manufacturing cost (y-axis, J_2), using abrasive water jet cutting as the reference process. The simplest design is made up of a single bar. It is the cheapest to manufacture, but also uses the material in the least efficient way. The 17-bar design on the other hand has both high performance (= small displacement under load) and high manufacturing cost due to its complexity. This complexity manifests itself through higher cutting length and sharper cutting radii. Along both objectives there is a relationship of diminishing returns. For many applications the design near the “knee” of the curve in Fig.9 will be most attractive. That “knee”, however, cannot be found if artificial constraints are introduced ab initio as is the case in many single objective optimization formulations.

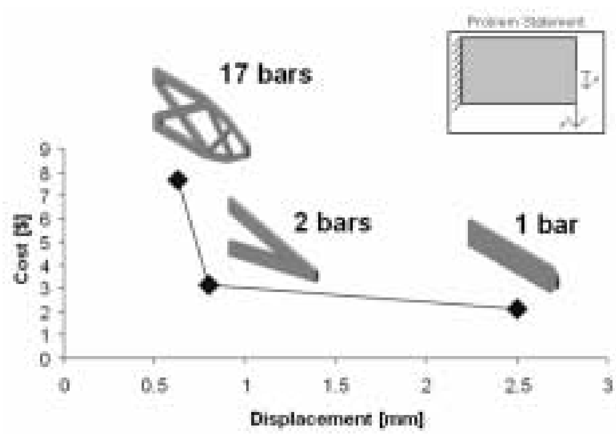

Fig.9: Tradeoff between structural performance and cost

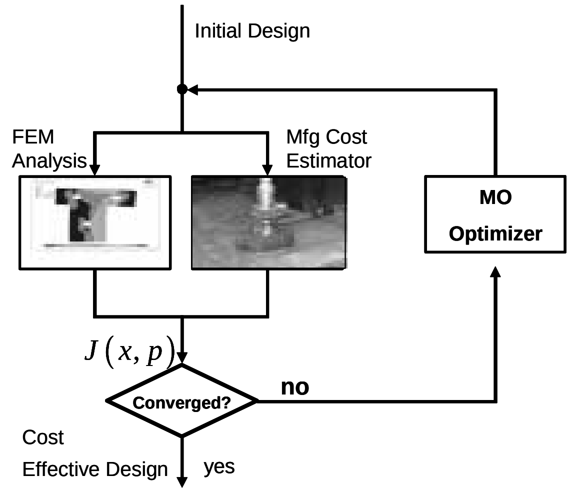

Fig.10: Framework for performance/cost optimization

A simplified version of a concurrent multiobjective optimization framework for structural performance and manufacturing cost is depicted in Fig.10. The idea is that topology/shape optimization are combined with FEM evaluation of structural performance and cost estimation. More details on initial work in this area are provided in (Kim, Nadir and de Weck 2004). One of the most interesting conclusions is that high-fidelity manufacturing cost models can be very non-linear. This is particularly true when features are introduced that push a design close to the feasibility constraints of a particular manufacturing process. These non-linearities cause convergence and stability challenges in the optimization loop. Conventional wisdom approximates cost models with low order cost estimation relationships (CERs); a practice which we found to be far removed from industrial reality.

5.2 Adaptivity

All multiobjective methods discussed thus far attempt to find the Pareto Front of a system in a pre-determined fashion. In the case of the traditional weighted sum method (Eq. 7) the weights λ_i are defined a-priori. This can result in wasted computational effort in many cases. Fig. 5 shows that despite equal spacing of weights, many points are nearly identical in the objective space for our numerical example from Eq. (2,3).

A new set of adaptive methods are currently being developed, whereby a few points are found on (or near) the Pareto front initially and this information is subsequently used to invest computational effort in unexplored, or poorly represented regions of the objective space. The Adaptive Weighted Sum (AWS) method is one such approach (de Weck and Kim 2004a, 2004b). The following paragraph demonstrates the use of the AWS method for multi-objective optimization of the classical three-bar truss problem first presented by (Koski 1985). Figure 11 illustrates the problem and shows the values of the parameters used. A horizontal load and a vertical load are applied at Point P, and the objective functions are the total volume of the truss members and the displacement of point P. The mathematical problem statement is

$$\begin{aligned}
&\text{minimize} \quad \begin{bmatrix} \text{volume}\ (\mathbf{A}) \\ \Delta(\mathbf{A}) \end{bmatrix} \\
&\text{subject to} \quad \sigma_{\text{lower limit}} \le \sigma_i \le \sigma_{\text{upper limit}} \quad, i = 1,2,3 \\
&\qquad\qquad\quad A_{\text{lower limit}} \le A_i \le A_{\text{upper limit}} \quad, i = 1,2,3 \\
&\text{where} \ \ \Delta = 0.25\delta_1 + 0.75\delta_2 \\
&\qquad\quad \text{and} \ \ \mathbf{A} = [A_1\ A_2\ A_3].
\end{aligned} \tag{17}$$

The Pareto front for this example is non-convex, and the Pareto line is separated into two regions by a segment of dominated solutions, as shown in Fig. 11. The adaptive weighted sum method with an offset δ_J of 0.1 is used. The optimization history is shown in the figure. The adaptive weighted sum method converges in three phases, and the solutions are quite evenly distributed. Note, that there is no solution obtained in the non-Pareto region, without using a Pareto filter. The parameter δ_J is used to tune the desired density of Pareto points generated by the algorithm.

The adaptive weighted sum method effectively approximates the Pareto front by gradually increasing the number of solutions on the front. In that sense it gradually "learns" the shape of the Pareto front and concentrates computational effort where new information can be gained most effectively. This is in contrast to other Pareto generation methods such as traditional weighted sum or NBI, which generally explore the Pareto front in a predetermined fashion. Because it adaptively determines where to divide further, the adaptive weighted sum method produces well-distributed solutions. In addition, performing optimization only in feasible regions by imposing additional inequality constraints enables the method to find Pareto solutions in non-convex regions. Because the feasible region includes only the regions of non-dominated solutions, it automatically neglects non-Pareto optimal solutions. It is potentially more robust in finding optimal solutions than other methods where equality constraints are applied.

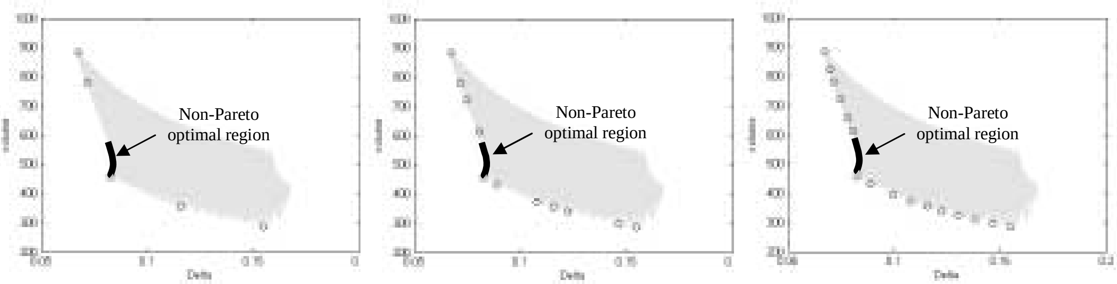

Fig. 11: Optimization history by the adaptive weighted sum method. $\delta_J = 0.1$.

There are clear indications that research in adaptive multiobjective methods will continue in the coming years.

6. Emerging Applications for Complex Systems

So far, we have discussed Multiobjective Optimization under the assumption that its purpose is simply to identify a set of Pareto-optimal, or at least non-dominated solutions, among which a final alternative will be chosen. We have not discussed in detail how such a final choice would be made. This aspect is treated extensively in the field of decision theory.

There are however, a number of very interesting and emergent uses of Pareto Frontiers, which have not yet been discussed. These emerging applications of Pareto analysis involve complex systems[4], as well as lifecycle considerations. Many such lifecycle considerations are summarized under the term "Illities": reliability, maintainability, reconfigurability or extensibility, among others. These are also "performance" objectives; however, they often reveal themselves only over time and not necessarily at the time of manufacture or first usage of a system. Pareto frontiers can be very helpful in understanding, quantifying and visualizing the relationship between these lifecycle objectives and the short-term objectives of performance, cost and risk (Fig.1) which are manifest during product development.

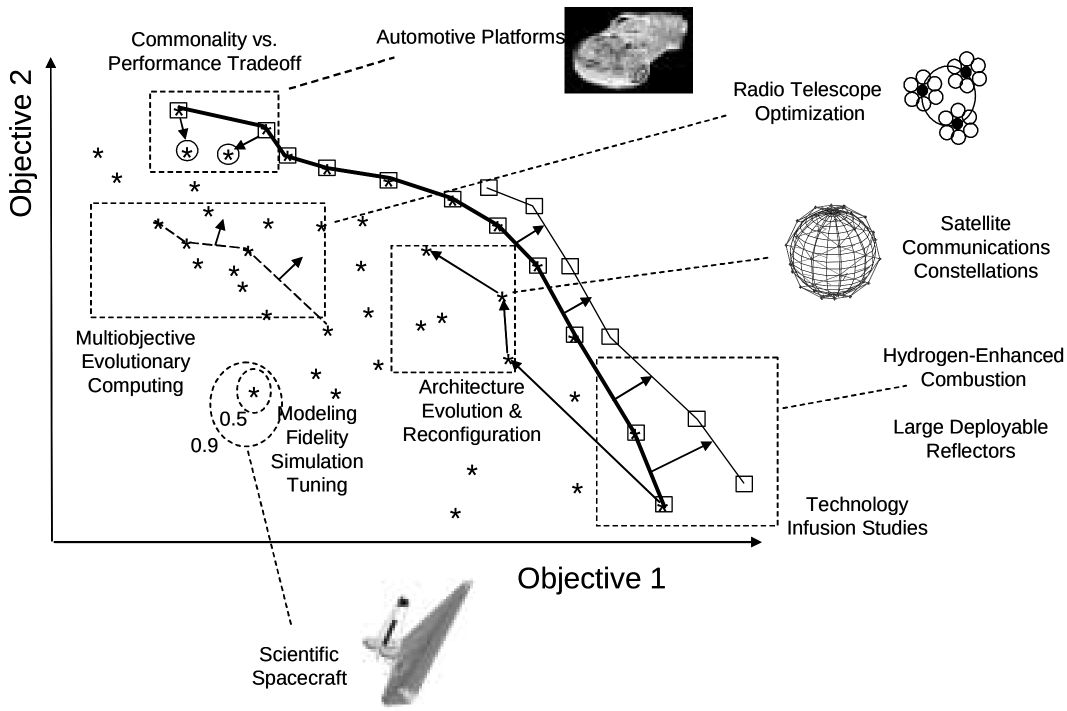

Fig.12: Emerging Applications of MOO and Pareto Frontiers to Complex Systems and Lifecycle Engineering

Fig.12 shows an overview of such emergent applications of Pareto frontiers to Complex Systems and Lifecycle Engineering. In product platforms (e.g. for automobiles) tradeoffs have to be resolved between commonality and performance of variants in the same product family. MOO was applied to the optimization of large future radio telescope arrays (Cohanim, Hewitt and de Weck 2004). It was found that different topologies dominate different regions of the Pareto front. Multiobjective optimization was also applied to the concept of staged deployment of satellite constellations (de Weck, de Neufville and Chaize 2004). It was found that embedding flexibility in a system to allow for future expansion leads to sub-optimality and short term (cost and mass) penalties. The benefits of such flexibility over the lifetime of the system, however, can significantly outweigh these initial expenses if future demand for the system is uncertain. Finally, it is suggested that Pareto Fronts are potentially more effective than S-curve models at quantifying

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[4] We define complex systems as those that have > 7^3 components and are relatively long-lived. Single components or simple structures such as the three-bar truss discussed above are not complex systems.

the performance/cost effects of technology infusion and technology obsolescence studies applied to complex systems (de Weck, Chang 2003). The infusion of new technologies often leads to the removal of existing constraints, see Eq. (1, 16), which in turns can cause shifts in existing Pareto Frontiers. These Pareto shifts can be quantified and used as an indication of likely success of new technologies or designs over the status-quo.

Conclusions

A multiobjective (vector) optimization problem generally has more than one solution. As was shown in this paper we are typically interested in finding a z-dimensional Pareto front or a set of non-dominated solutions to solve a design optimization problem. Two fundamental approaches to MOO problems can be distinguished: Scalarization Methods for multiple objectives (e.g. Utility Theory) and Pareto Approaches with a-posteriori preference expression. Various methods for computing Pareto Fronts have been developed in the last 30 years to support engineering design and multicriteria decision making: Weighted Sum Approach (and variants), Design Space Exploration + Pareto Filtering, Normal Boundary Intersection (NBI), Multiobjective Heuristic Algorithms (GA and SA), among others. None of these methods are perfect and selecting among them depends on the requirements of a particular design situation. Resolving tradeoffs is an essential part of system optimization and design and this will remain so in the future.

Future trends include a stronger emphasis on manufacturing cost as well as the emergence of adaptive algorithms for multiobjective optimization and visualization. There are many applications for Pareto analysis, particularly for complex systems and lifecycle engineering. We have barely scratched the surface in terms of understanding the relationship between short term performance/cost/risk criteria and long term system properties such as: maintainability, reconfigurability or extensibility.

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