Spatial Nyquist Fidelity Method for Structural Models of Opto-Mechanical Systems

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ABSTRACT

Simulation models of new opto-mechanical systems are often based on engineering experience with older, potentially dissimilar systems. This can result in inaccuracies in the model prediction. A method is needed to gauge the fidelity of new system models in the initial design phases, often in the absence of hardware data. The Nyquist criterion is used to develop a quantitative measure of model fidelity, called the Nyquist fidelity metric. The spatial Nyquist fidelity method is presented which uses the Nyquist fidelity metric to both assess the fidelity of existing complex models and to synthesize new multi-component models starting from architectural considerations such as geometric and material properties of the system. This method also estimates the error bound on the output figures of merit based on the fidelity levels and sensitivity analysis. The Nyquist fidelity method is applied to the Modular Optical Space Telescope (MOST), the Thirty Meter Telescope, and the Stratospheric Observatory for Infrared Astronomy. It is shown in the MOST case study that the Nyquist fidelity method provides a 40% improvement in computational time while assuring less than 5% modal frequency error, and less than 2.2% error in the output figure of merit.

Keywords: fidelity, Nyquist, spatial frequency, integrated modeling

1. INTRODUCTION

Modeling and simulation using integrated opto-structural models are important during the conceptual and preliminary design phases of complex telescope systems. The dynamic structural behavior of these telescopes is critical to overall performance. Due to the expense and time constraints of hardware modeling and the difficulty of test and validation, we rely on simulation models to provide performance estimates during the conceptual and preliminary design phases. Such models are often qualitatively described as high-, mid-, or low-fidelity.

This paper provides a fidelity metric based on the Nyquist criterion that quantitatively measures the level of fidelity in telescope structural models. Here, fidelity describes the ability of the model to accurately estimate a specific output figure of merit, and the fidelity metric reflects this quantitatively. The fidelity metric is defined for beams and plates since beams and plates are commonly used components in conceptual and preliminary design phase simulation models of many dynamic structures. Using the fidelity metric in model construction affords computational and development time savings by enabling the modeler to choose the level of model accuracy *a priori*.

In Sections 2 and 3 the fidelity metric is defined for beam and plate component models. Section 4 demonstrates how the Nyquist fidelity metric is used to assess the fidelity level of existing complex telescope models of the Thirty Meter Telescope (TMT) and Stratospheric Observatory for Infrared Astronomy (SOFIA). The fidelity metric is then used as part of the Nyquist fidelity method to size the fidelity level of the Modular Optical Spacecraft Telescope (MOST) model in Section 4. Section 4 performs a disturbance analysis on MOST and compares the accuracy and the computation time of the Nyquist fidelity method to the model reduction method.

2. FIDELITY METRIC

The fidelity metric for structural models is based on the Nyquist principle. The transverse deformation of a structure can be likened to a time domain signal. The finite element model nodes along the structure can be likened to signal sampling points. A rule-of-thumb in modeling such structures is to apply the Nyquist theorem and to make sure that there are enough sample points (nodes) along the finite element structure in order to capture a sufficiently high spatial frequency

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in the model[1]. This work formalizes this rule and quantifies fidelity of structural component models based on the satisfaction of the Nyquist requirement. The fidelity metric is as follows,

$$F_{\lambda} = \frac{n}{n_{Ny}} \tag{1}$$

where n is the number of nodes per unit length along the dimension of a structural component and n_{Ny} is the number of nodes per unit length required by the Nyquist criterion[2]. The term, n_{Ny}, is calculated as twice the highest spatial frequency of interest,

$$n_{Ny} = 2\varpi_{\max} \tag{2}$$

The highest spatial frequency of interest is calculated from the highest time domain frequency of interest using wave propagation theory[3,4]. The highest time domain frequency of interest is obtained from system disturbance spectra. These equations show the basis of the fidelity metric.

2.1 Beam fidelity

The beam fidelity metric is previously defined in Eqns. 1 and 2, where n is the number of nodes per unit length along the beam length and n_{Ny} is the number of nodes per unit length required by the Nyquist criterion.

The highest spatial frequency of interest, $\varpi_{\max}$, is related to the highest time-domain frequency, ω, using the wave propagation equations for beams

$$\varpi = \frac{k}{2\pi} \tag{3}$$

where ϖ is a spatial frequency and k is the wavenumber[3,4]

$$k = \pm\left[\frac{\omega^2 \rho A}{EI}\right]^{1/4}, \tag{4}$$

where ρ is the volumetric mass density, A is the beam's cross-sectional area, E is the Young's modulus and I is the area moment of inertia. These equations describe the relationship between modal frequencies in the time and spatial domains. It should be noted that these equations are only valid for undamped, homogenous, isotropic beams with constant cross sections[5].

To define the relationship between fidelity and modal accuracy, several finite element beam models are created and their modal accuracy as a function of fidelity is observed. A 2-D pinned beam, a 2-D cantilever beam, and a 2-D free-free beam, each of unit length 1, are modeled as aluminum with a solid, circular, 10 centimeter diameter cross section using simple beam elements. Twenty models are constructed for each boundary condition, each containing the corresponding number of evenly spaced beam elements (one through twenty). In addition to varying the number of nodes for each beam, the material and geometric properties are varied one at a time. The results for the pinned beam can be seen in Fig. 1.

The plots for the free and cantilever beams are similar[6]. This shows a remarkable result: the fidelity metric is strongly related to modal frequency error across varying properties and across modes. Therefore, the modal frequency error can be specified a priori and the corresponding fidelity level would suggest a mesh density that would achieve the desired error for all modes up to a certain frequency.

Below a fidelity level of 1.0 (vertical line marked in Fig. 1), the Nyquist theorem states that the models do not contain sufficient fidelity to capture the frequencies of each mode. Below a modal frequency error of 0.01% (horizontal line in plots), there are not enough significant figures in the modal frequency estimates to know the error to this precision. This is due to the number of significant figures used in the analytical and FE solution sequences. In the FE solution, only 4 significant figures are used for the cross sectional area and moment of inertia properties. Therefore, the modal frequency error metric is precise only to the fourth significant digit, or 0.01%.

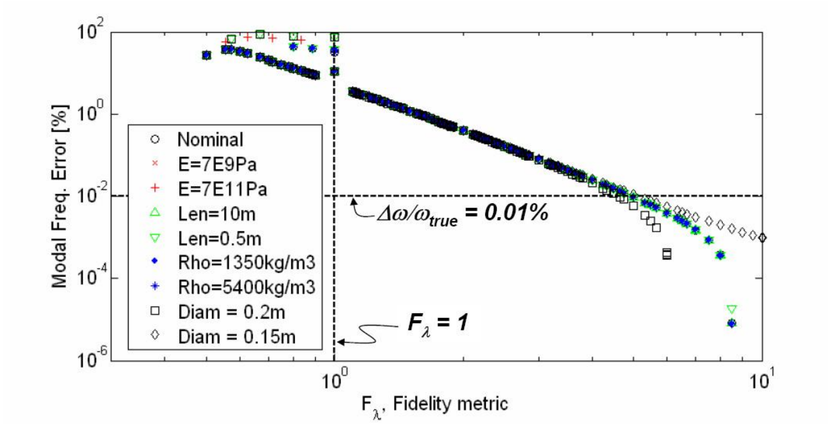


Figure 1: Pinned beam fidelity relationship

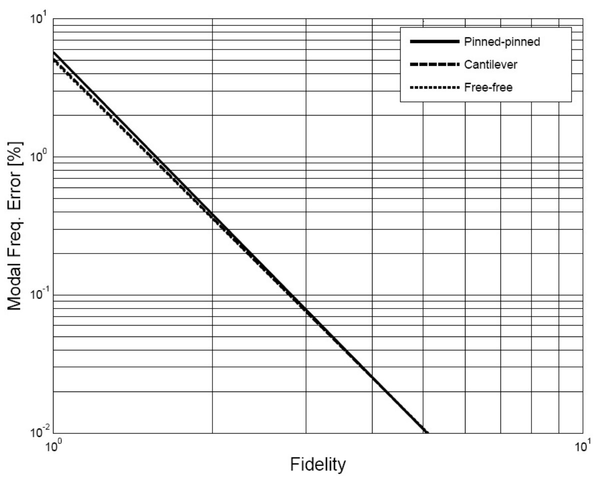


Figure 2: Power law curve fit

A curve fit is applied to the data shown in Fig. 1 for each boundary condition. A power law, often present in physical systems, is fit to the data within the bounds described above. The curve fit gives the modal frequency error as a function of fidelity and is shown in Fig. 2. The curve fit for the pinned-pinned case is the most conservative relationship because for a specified modal frequency error, the highest level of fidelity over the three boundary conditions is indicated. Therefore these coefficients are used to define the **Generalized Beam Equation** as

$$\log_{10}\delta\omega = a\log_{10} F_\lambda + b, \tag{5}$$

where a = 0.7579 and b = -3.907. Note that this relationship is only valid for fidelity levels greater than 1.0.

2.2 Plate fidelity

The fidelity metric for a generalized plate is based on the propagation of a transverse flexural wave through a thin plate. It is defined as

$$F_\lambda = \frac{\left(\sqrt{\#nodes/m^2}\right)_{FEM}}{\varpi_{Ny}}, \tag{6}$$

where the denominator is the areal node density of the plate model and the numerator is the Nyquist spatial frequency, which is twice the highest spatial frequency of interest,

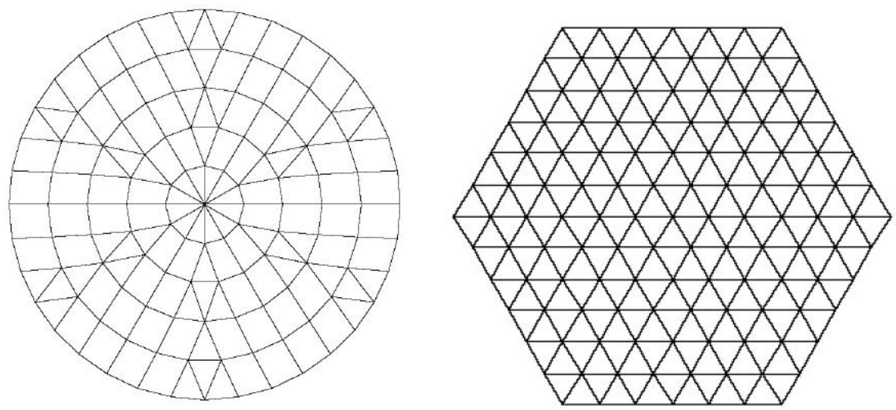
$$\varpi_{Ny} = 2\varpi \tag{7}$$

The highest spatial frequency of interest is related to the highest time-domain frequency of interest using wave propagation methods as in the beam example. For undamped plates, the wave motion can be described using a wavenumber of

$$k = \sqrt{\omega}\left[\frac{\rho h}{D}\right]^{1/4} \tag{8}$$

where ω is the time-domain frequency, ρ is the volumetric mass density, *h* is the plate thickness and *D* is the flexural rigidity[3].

Two different plates with two FE meshes are examined in this section. They can be seen in Fig. 3. A convenient way to mesh a circular plate is with a combination of quadrilateral and triangular plate elements in concentric rings as shown in Fig. 3(a) which consists of 5 rings of similarly-sized elements. A hexagonal plate can be meshed with uniform sized triangular elements as in Fig. 3(b), where there are 6 element rings. In the same way as the beam and plate fidelity metrics, the mesh densities of these plates are varied, thereby varying the fidelity and the modal frequency error is observed. Both plates are evaluated with free and pinned boundary conditions.



(a) Circular plate (b) Hexagonal plate

Figure 3: Plate finite element meshes

The relationship between fidelity and modal accuracy for each plate for the free boundary condition are shown in Figs. 4 and 5. The modal frequency error is calculated by taking the "truth" model as a highly meshed plate. The 10 solid lines represent the first ten flexural modes. In addition, boundary lines are drawn at a fidelity level of 1.0 and a modal frequency error of 0.01%, defining the minimum fidelity required and the precision of the calculation, respectively. While the data points do not fall roughly along a straight line, they do seem to have a upper boundary on the maximum fidelity required for a given modal frequency error.

In both of the above plots this upper boundary is also plotted in a thick dashed line. This relation provides a mostly conservative boundary on this data and is given as

$$\Delta\omega\left[\%\right] = 175 F_{\lambda}^{-2.2808} \tag{9}$$

The fidelity relation shown in Eqn. 9 is called the **Generalized Plate Relation** and is used for plates of any shape. It is valid for fidelity levels greater than 1.0 and less than 72.5 (this corresponds to a modal frequency error of 0.01%). A recommended minimum fidelity for plates, though, is 4.0 to ensure that the displacements associated with the modeshapes are represented accurately. It is important to accurately represent displacements, since these displacements are in many cases used to calculate the output figures of merit.

2.3 Output error estimate

In addition to the modal frequency error estimates, sensitivity analysis techniques can be used to calculate an *a posteriori* estimate of the error bound on the output figure of merit (OFM). For this analysis, the OFM must be a linear combination of the outputs of the system state space integrated model.

The error bound of the OFM can be estimated by multiplying the sensitivity by the modal frequency error specified, $\Delta\omega_k$, and then summing over the modal frequencies,

$$\frac{\Delta\sigma_z}{\sigma_z} = \sum_{k=1}^{m} |S_k| \Delta\omega_k \,, \tag{10}$$

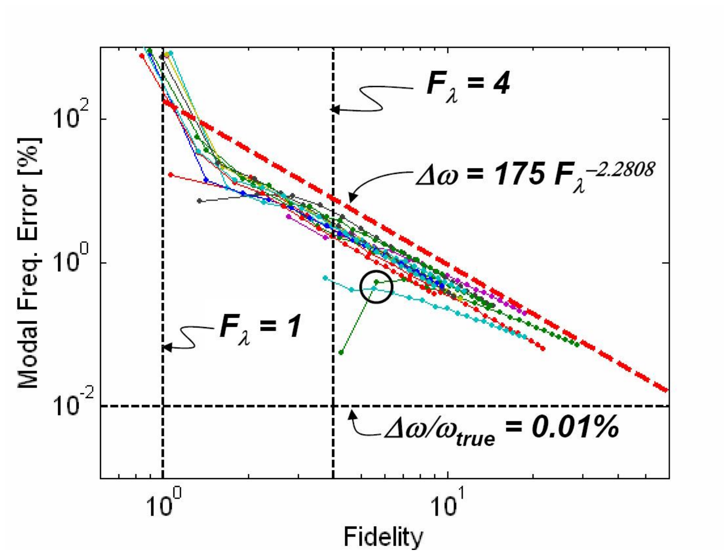


Figure 4: Fidelity relationship for a free circular plate

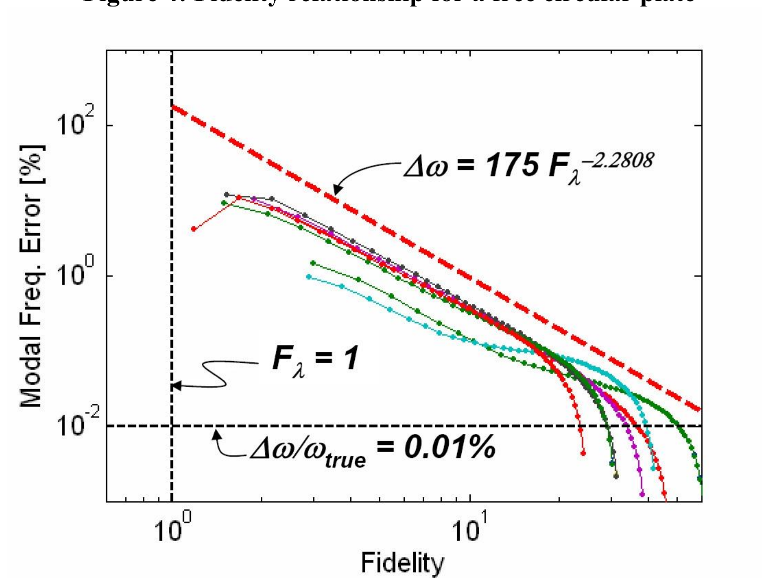


Figure 5: Fidelity relationship for a free hexagonal plate

where S_k is the sensitivity of the output figure of merit with respect to changes in the kth modal frequency and m is the number of system modal frequencies below the frequency level of interest. The sensitivity values can be calculated using methods shown in Uebelhart[7].

3. FIDELITY ASSESSMENTS

In this section two case studies demonstrating the application of the Nyquist fidelity metric to real world complex systems are shown. These are the Thirty Meter Telescope (TMT) and the Stratospheric Observatory for Infrared Astronomy (SOFIA). Both of these programs are established and structural models exist for each. The fidelity of these existing models is assessed and suggestions are made to improve the fidelity of the model.

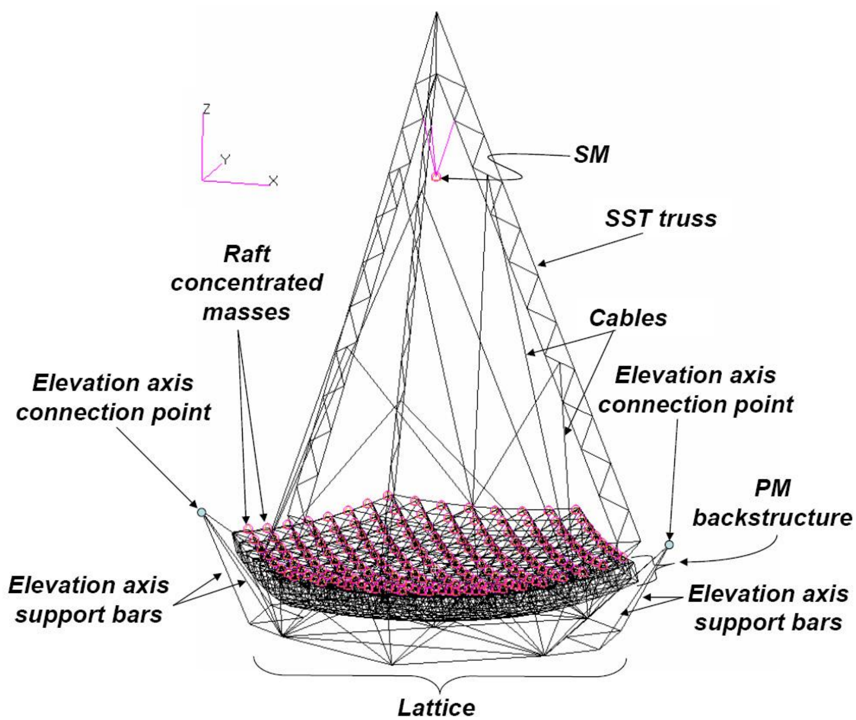


Figure 6: TMT Structural FEM

3.1 TMT

The Thirty Meter Telescope is a ground-based optical/infrared astronomical research observatory scheduled for completion in 2015. It employs adaptive optics to provide diffraction limited performance over 1-2 arc minute fields with sensitivities that will enable imaging of some of the faintest objects in the universe[8]. The model examined in this section is the structural TMT model developed by MIT in 2004 which represents a conceptual design of TMT.

For the fidelity assessment, the structural model of the optical telescope assembly (OTA) and part of the elevation structure are examined. The OTA consists of the primary and secondary mirrors, the secondary tower structure (SST), and the primary mirror backstructure. The elevation structure consists of a lattice structure and elevation axis connection bars. The wireframe FEM of the OTA and elevation structure can be seen in Fig. 6.

Since TMT is a ground based observatory, the major disturbance force encountered is from wind. Four disturbances are defined: (1) forces on the primary mirror from the slit (M1s), (2) forces on the primary mirror from the vents, (3) forces on the secondary mirror from the slit, and (4) forces on the primary mirror due to the shear layer mode. The upper disturbance frequency in all four cases is 20 Hz. This is the upper maximum frequency considered for the Nyquist fidelity metric.

The model shown in Fig. 6 contains 2335 beams. They can be separated into five groups representing five different beam properties: the PM backstructure, the raft support bars, the SST, the bars connected to the elevation axis points, and the lattice structure. Using the fidelity metric and the upper disturbance frequency of 20 Hz, the fidelity of each beam can be calculated. Then, using the generalized beam equation, Eqn. (5), the modal frequency error is estimated. The results are shown in Fig. 7. The data point in Fig. 7 to the left of the minimum fidelity requirement represents two lattice bars that have fidelity levels less than 1.0. Besides the two offending lattice bars, the next lowest fidelity level across all the bars is 1.30, present in the lattice and SST truss members. This corresponds to a modal frequency error of 2.08%. Therefore, if the two offending lattice bars where sufficiently meshed to have a fidelity greater than or equal to 1.30, then the expected modal frequency error in the model due to under-meshing would be 2.08%. This is used in the next section to estimate OFM error.

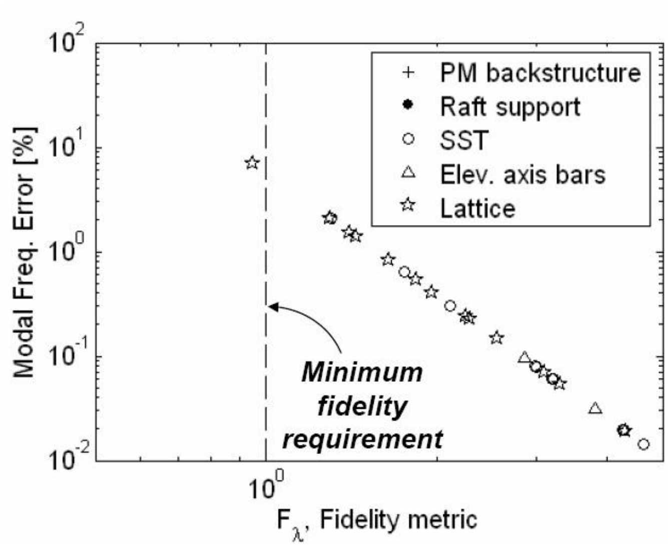


Figure 7: Modal frequency error of TMT beams*

The output figures of merit for this model are image motion, *IM*, and image quality, *IQ*, given by

$$IM = \sqrt{\sum_{j=1}^{4}\sum_{i=2}^{3}\sigma_{i,j}^{\ 2}} \tag{11}$$

$$IQ = \sqrt{\sum_{j=1}^{4}\sum_{i=4}^{9}\sigma_{i,j}^{\ 2}} \tag{12}$$

where σ_{ij} is the *ith* RMS Zernike coefficient resulting from the *jth* disturbance source. There are four disturbance sources, listed above. The image motion is similar to line-of-sight jitter and is determined by the second and third Zernikes which represent tip and tilt of the image. The image quality is determined by the higher order Zernikes. The Zernikes, in turn, are determined by mirror displacements from the structural model.

To estimate the errors in the image motion and image quality, first the nominal performances are evaluated. These are

$$\mathrm{IM} = 2.6126\mathrm{E}-5[\mathrm{m}]$$

$$\mathrm{IQ} = 6.3102\mathrm{E}-7[\mathrm{m}]$$

This is based on 100 modes extracted from the FEM. In fact, the highest mode extracted is at 17.08 Hz, which falls below the upper disturbance frequency of 20 Hz. Adjusting the analysis such that all modes at or below 20 Hz are extracted (total of 237 modes), gives slightly lower nominal performances of

$$\mathrm{IM} = 2.6124\mathrm{E}-5[\mathrm{m}]$$

$$\mathrm{IQ} = 6.2901\mathrm{E}-7[\mathrm{m}]$$

Next, a sensitivity analysis is performed, which yields the sensitivity of the Zernike RMS values with respect to each system mode for each disturbance source. With a 2.08% MFE, the OFM error estimate bounds are:

* Note that beam and plate elements with fidelities larger than 20 are not shown here

$$IM = -473.45\%$$

$$IQ = -402.99\%$$

These large numbers are due to the high sensitivity of these OFM to the structural modes. Recall that this error measure does not indicate how much error is present in the OFM, rather it is an estimate of the error bound on the OFM due to under-meshing. Therefore the actual error in the OFM due to under-meshing is equal to or below these values. As a comparison, were all the beams meshed sufficiently fine to produce a 0.01% modal frequency error, the OFM error estimates would be

$$IM = 2.2762\%$$

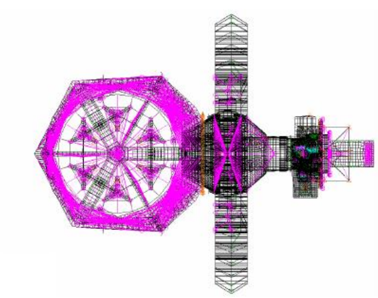
$$IQ = 1.9374\%,$$

which is a significant improvement in the error bound estimate.

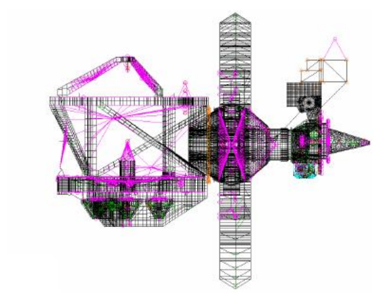
3.2 SOFIA

The Stratospheric Observatory for Infrared Astronomy (SOFIA) is an airborne telescope developed as a cooperative effort between NASA and the DLR Deutsches Zentrum fuer Luft- und Raumfahrt (German Aerospace Agency) and supported by the Universities Space Research Association (USRA) and the Deutsches SOFIA Institut (DSI). SOFIA has a parabolic 2.7 m diameter primary mirror and is capable of gathering science light from the visible to the submillimeter (far-infrared) range (0.3 μm – 1600 μm) with pointing accuracies less than 1 arc second. SOFIA is housed in the aft section of a Boeing 747SP and will perform science observations while flying at 39,000 to 45,000 feet through a specially designed fuselage door.

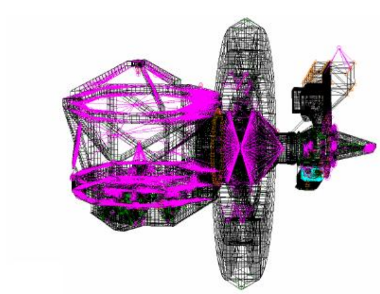
The overall geometric layout of the SOFIA structural model can be seen in the wireframe finite element model (FEMs) shown in Fig. 5-17. This model was first developed by the telescope's manufacturer, MAN AG of Mainz Germany, then converted to Nastran by Orbital Sciences Corporation[9]. It contains over 70,000 elements and over 331,000 degrees of freedom. The main challenges for SOFIA include counteracting the wind disturbance entering from the fuselage door and the aircraft disturbance entering through the bearing support system. Based on the frequency content of these disturbances, the highest time-domain frequency of interest is chosen to be 100 Hz.



(a) Top view



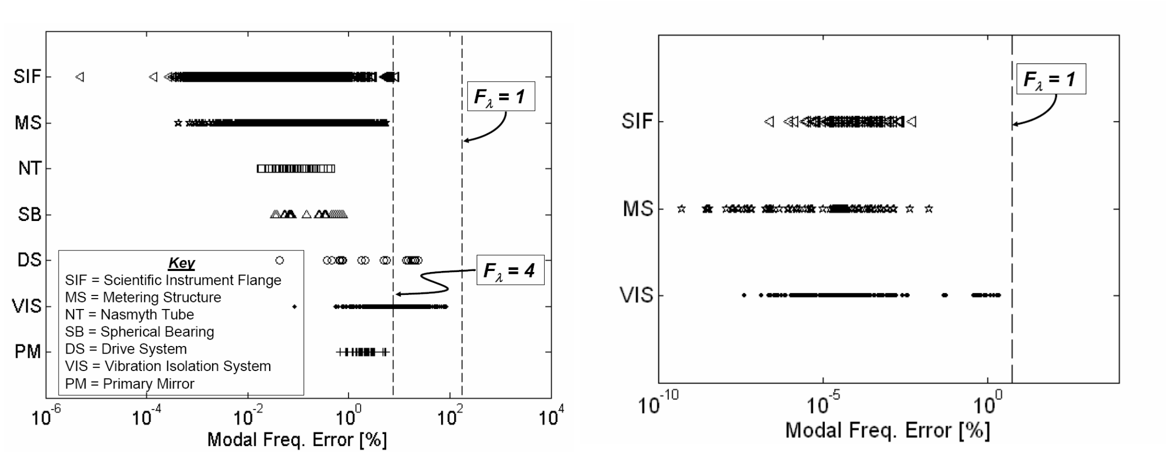
(b) Side view



(c) Isometric view

Figure 8: SOFIA finite element model

The output figure of merit of this model is the RMS image motion, the output of the integrated model (IM). The fidelity of each of the approximately 75,0000 individual beam and plate elements in the SOFIA model is examined. The results can be seen in Figs. 8 (a) and (b)†. They are broken up according to the subsystems listed in the legend. The vibration isolation system, the drive system, and the science instrument flange have plate elements with fidelities below the recommended minimum level of four. The rest of the plates meet or exceed this requirement. All of the beams in this model meet the minimum fidelity requirement of one.



(a) MFE for SOFIA plates (b) MFE for SOFIA beams.

Figure 9: Modal frequency error for SOFIA plates and beams

The fidelity assessment of the models of the major SOFIA subsystems shown above highlight areas such as the plates in the vibration isolation and drive systems that require further mesh refinement to meet Nyquist requirements and to provide an estimate for the system's modal frequency error.

4. FIDELITY METHOD

The fidelity metric can also be used to synthesize telescope models given a geometric and material design. Starting with the geometric properties, material properties, highest frequency level of interest, and chosen modal frequency error, the corresponding finite element mesh size can be calculated. Then, after the model is built and the sensitivity analysis is performed, an error estimate on the output figure of merit is calculated. If the OFM estimated error is too high, a lower MFE can be chosen, and the model can be rebuilt until an adequate MFE is obtained[6].

This method is applied to the MOST telescope design, shown in Figure 10. MOST is a spacecraft telescope designed to image targets in the optical and infrared wavelengths. The nominal configuration of MOST is a Cassegrain telescope with a 3 m diameter primary mirror and a tripod support for the secondary mirror support tower. There are two isolators not explicitly shown in this figure, one between the reaction wheel assembly (RWA) and the bus and one between the bus and the OTA. MOST is currently in the conceptual design phase.

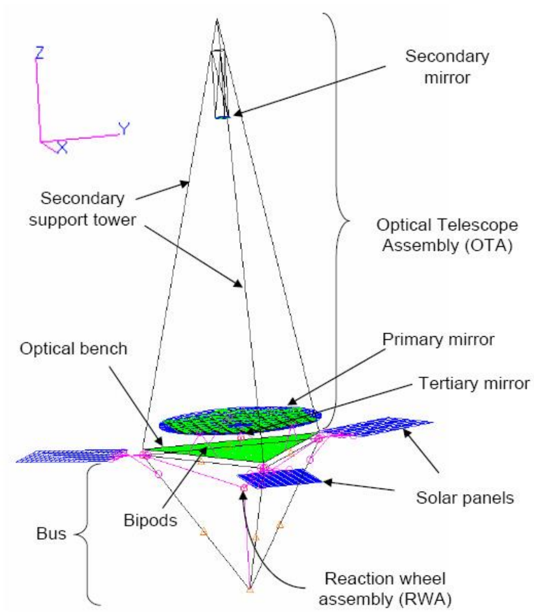


Figure 10: MOST nominal configuration

† Note that beam and plate elements with fidelities larger than 20 are not shown here

The output figures of merit for this design are chosen to be RMS line-of-sight jitter in the x and y directions on the focal plane (LOS_x, LOS_y). The calculations for LOS in the x- and y-directions in the image plane can be found in a memo by Perrygo and Burg[10] and are,

$$LOS_x = -\frac{1}{f_1}\delta_{P_y} + \frac{(M-1)}{Mf_1}\delta_{S_y} + \frac{1}{Mf_1}\delta_{T_y} + 2\alpha_{P_x} - \frac{2}{M+1}\alpha_{S_x} - \frac{2}{M+1}\alpha_{T_x}$$
$$LOS_y = \frac{1}{f_1}\delta_{P_x} - \frac{(M-1)}{Mf_1}\delta_{S_x} - \frac{1}{Mf_1}\delta_{T_x} + 2\alpha_{P_y} - \frac{2}{M+1}\alpha_{S_y} - \frac{2}{M+1}\alpha_{T_y} \quad (13)$$

where δ and α are the displacements and rotations, respectively, of the primary (P), secondary (S), and tertiary (T) mirror center points in or about the x- and y-directions in the image plane. The optical parameters included are the focal length of the primary mirror, f_1, and the secondary mirror magnification, *M.* LOS is an angular measurement. The main disturbance source is considered to be the imbalance of the reaction wheels, and the highest frequency associated with it is 100 Hz.

Two levels of modal frequency error (MFE) are chosen for comparison purposes: 1% and 5% MFE. The generalized beam and plate equations are first used to calculate the fidelity levels. Then, the fidelity levels are translated into mesh density using the definition of the fidelity metric. There are four different beam lengths and four different plate models present in the MOST model. The resulting mesh densities for each MFE level can be seen in Tables 1 and 2.

Table 1: Mesh sizing for MOST beams

MFE	Fidelity	7.81 m beam # nodes	1.21 m beam # nodes	1.18 m beam # nodes	0.25 m beam # nodes
1%	1.56	10	4	4	2
5%	1.04	7	3	3	2

Table 2: Mesh sizing for MOST plates

MFE	Fidelity	Primary mirror nodes/m^2	Secondary mirror nodes/m^2	Optical bench nodes/m^2	Solar panel nodes/m^2
1%	9.63	1315	13	193	2
5%	4.75	321	3	47	1

The mesh sizes summarized above are used as guidelines in complex systems. Practical considerations such as connectivity and element topology are taken into consideration when sizing the final mesh. After the final mesh is constructed, the nominal OFM are calculated, these values are shown in Table 3. Next using sensitivity analysis results and Eqn. 10, OFM error estimates are calculated, also shown in Table 3.

Table 3: OFM nominal values and error bounds

MFE	LOS_x [mas]	LOS_y [mas]	LOS_x error est. [%]	LOS_y error est. [%]
1%	1.6491	3.5363	2.16	2.10
5%	1.7376	3.5146	10.78	10.05

Finally a comparison is made between the Nyquist fidelity method of model creation and the model reduction method. The model reduction method begins with the MOST nominal model and retains a certain number of system modes. It is also known as balanced truncation. Further information on the model reduction method can be found in Moore[11] and Gawronski[12].

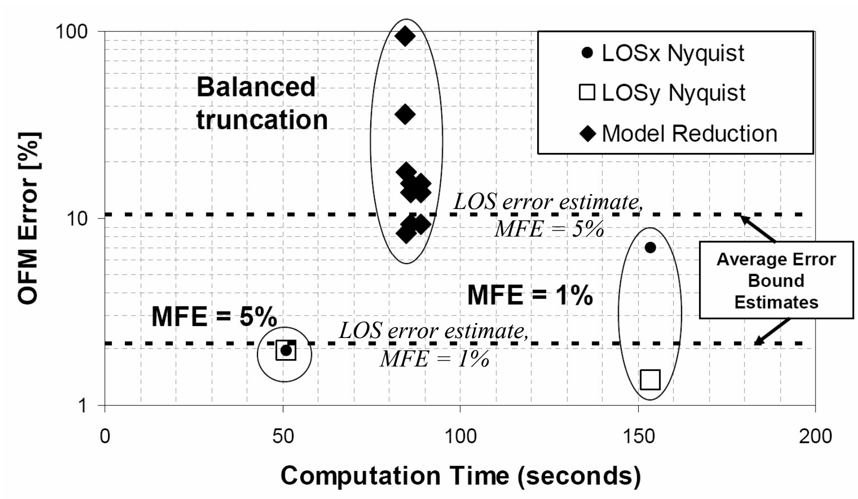


Figure 11: Calculation time vs. OFM error for MOST

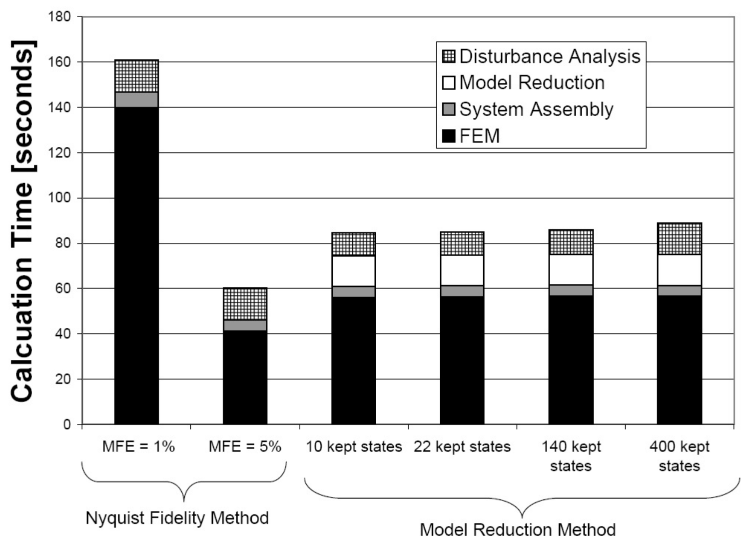


Figure 12: Calculation time comparison chart for MOST

The accuracy of the OFM for both methods can be seen plotted against computation times in Fig. 11. The dashed lines represent the error bound estimates listed in Table 3. It can be seen that the Nyquist fidelity method takes about 80% longer for the MFE = 1% case than the model reduction method. This is driven by the high fidelity requirements of the primary mirror and optical bench. However, the accuracy of the Nyquist method is greater. In the MFE = 5% case, about 40% less computational time is required for the Nyquist method than the model reduction method. Furthermore, since the accuracies in the Nyquist method are better than model reduction method, the model created using the Nyquist fidelity method is a more appropriate model overall. This plot illustrates that the Nyquist fidelity method provides better accuracy over model reduction and with lower computational costs in some cases.

It also shows the violation of the estimated error bound for the LOS_x for the MFE = 1% case. Note also that despite a lower MFE for this data point, the MFE = 5% case outperforms it for the LOS_x. This is most likely due to the numerical ill-conditioning of the integrated model.

The plot in Fig. 12 decomposes the computation time according to each process. It can be seen that the disturbance analysis for some of the model reduction method cases takes less time than for the Nyquist fidelity method. However, the finite element model (FEM) construction and system assembly take longer in the model reduction cases than in the Nyquist fidelity MFE = 5% case. Therefore, if the analysis calls for several disturbance analyses ("load cases") to be performed on one instantiation (configuration) of the system, balanced model reduction might be a better choice for a modeling method. However, if it is the case that several FEMs need to be constructed for analysis, as in the conceptual design phase where several different configurations are being investigated, than this shows that the Nyquist fidelity method can provide time savings.

5. SUMMARY

This paper defines a quantitative fidelity metric for telescope structural models. It also demonstrates the use of the Nyquist fidelity metric in assessing fidelity of two existing real-world telescope models, one for TMT and one for SOFIA. A method is also shown where structural model fidelity level is set *a priori* by calculating an appropriate finite element mesh size using the Nyquist fidelity metric for the MOST model. Future work includes extending the fidelity metric to other component types and to apply the Nyquist fidelity method to closed loop integrated modeling systems

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