A Mathematical Model for Interplanetary Logistics

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Introduction

This article demonstrates a methodology for designing and evaluating the operational planning for interplanetary exploration missions. A primary question for space exploration mission design is how to best design the logistics required to sustain the exploration initiative. Using terrestrial logistics modeling tools that have been extended to encompass the dynamics and requirements of space transportation, an architectural decision method has been created. The model presented in this article is capable of analyzing a variety of mission scenarios over an extended period of time with the goal of defining interesting mission architectures that enable space logistics. This model can be utilized to evaluate different logistics trades, such as a possible establishment of a push-pull boundary, which can aid in commodity pre-positioning. The model is demonstrated on an Apollo-style mission to both provide an example and validate the methodology.

The development of an interplanetary supply chain requires the unification of two traditionally separate communities: aerospace engineering and operations research. In order to create an effective means of communication between both communities, a distinct terminology has been developed and is detailed extensively in Section I. Specifically, the definition of the commodities or supplies, and the elements or physical containment and propulsion units used to transport the commodities are detailed. Furthermore, the network definition is presented as well as the definition and description of the time expanded network, which is the terrestrial modeling technique employed for the space logistics model. Section II describes the components of the interplanetary logistics problems. Section III presents the problem formulation and constraints. In Section IV a description of the optimization methodology developed to solve this problem is discussed. In Section V the problem formulation and solution methodology is applied for the example of an Apollo-style mission to both explain the implementation and validate the methodology presented. Section VI reviews the contributions of this article and describes continuing work in this area.

I. Problem Definition

The goal of the interplanetary logistics problem is to determine feasible mission architectures to satisfy the demand generated by the needs of exploration. The key concept of the interplanetary logistics problem is that the demand of crew, consumables, equipment and other exploration requirements at in-space locations drives the mission requirements. Therefore, the first required input for the interplanetary logistics problem is the definition of these supplies. For example, if the exploration mission is a sortie style mission to investigate a particular location, the demand might consist of a few crew members at a specific location and the supplies necessary to both support the crew and enable the exploration activities.

Given the demand of the mission, it is necessary to determine how and when the supplies on Earth will be transported to the in-space locations. As missions become more complex and evolve over a period of time, a solution may become less obvious. Since the goal is to minimize the cost of any mission, it is desirable to optimize the timing and method of transport of the supplies to in-space locations. Therefore, it is necessary to define all pathways and structures used for transport, and allow the optimizer to analyze the different architectures to select the best one.

Given this information, the interplanetary logistics problem can determine low cost mission architectures that satisfy the exploration demand. The solution generated will detail the scheduling and assignment of supplies to vehicles for in-space transport and launch scheduling requirements. More importantly, however, the output of this problem can be used to determine a push-pull boundary for the supplies, the potential of a specific location, either on a surface or in-space for storing supplies, benefits of in-situ resource utilization over multiple missions, or even the sensitivity of mission architectures to changes in vehicle parameters.

The first step in developing a model for interplanetary logistics is defining a concrete nomenclature that describes the components of the problem. The problem fundamentally consists of three components: the commodities or supplies that must be shipped to satisfy a mission demand, the elements or physical structures used to both hold and move the commodities, and the network or pathways the elements and commodities travel on. The following sub-sections define the parameters that describe each of these components.

Commodities

The goal of the space logistics project is to determine how to meet the demand for the exploration missions. As such, we are investigating how to optimally ship multiple types of commodities. For the purpose of the logistics problem, a commodity will be defined as a high-level aggregate of a type of supply, such as crew provisions. Thus, we will define a set of $k = 1, \ldots, K$ commodities, each with the following parameters:

- Denote the demand of each commodity as d^k.
- Denote the origin of each commodity as so^k.
- Define the destination of each commodity as sd^k.
- Define the availability interval of each commodity as $to^k = [sto^k, eto^k]$, where sto^k is the starting time of the interval and eto^k is the ending time of the interval.
- Define the delivery interval of each commodity as $td^k = [std^k, etd^k]$, where std^k is the starting time of the interval and etd^k is the ending time of the interval.
- Define the unit mass of each commodity as m^k when it arrives at the destination.
- Define the unit volume of each commodity as v^k when it arrives at the destination.
- Define the number of specified waiting sequences as nw^k.

By defining a waiting sequence as part of the commodity input, a number of wait arcs along the path can be specified, which allows on-route destinations to be designated. For each waiting arc sequence I where $0 < I < nw^k$ the following parameters must be defined:

- Define the static node of the wait sequence as sw_l^k.
- Define the required waiting time period as pw_l^k.
- Define the wait interval for each wait sequence as = $[tw_l^k, etw_l^k]$, where stw_l^k is the starting time of wait interval l of commodity k, etw_l^k is the ending time of wait interval l of commodity k, and $etw_l^k - stw_l^k \geq pw_l^k$.

It is important to note that in this model a crew member is treated as a commodity. In practice crewed missions are treated differently during mission planning: however, for the purposes of the architectural design tool created by this model, crew can be considered a commodity with highly restrictive parameter values. By narrowing the availability and delivery windows for a crew commodity, the feasible shipment pathways are limited and reasonable architectures for crewed flights can be obtained.

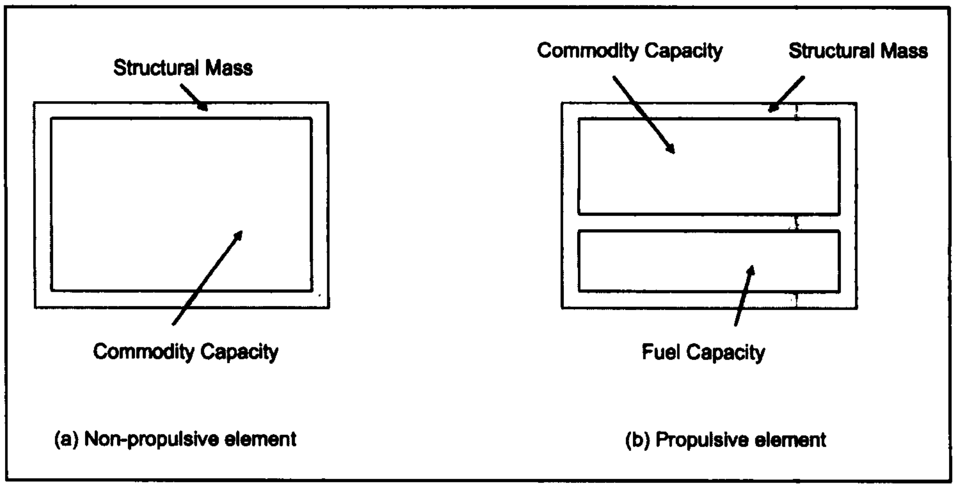

Figure 1
Element Representation

Elements

In order to ship the commodities from the origin to the destination locations, we require 'containers' to both hold the commodities and provide propulsion to move the mass through space. These components can be abstracted to a single definition of an element. Elements are physical, indivisible functional units that transport the commodities from origin to destination. An element is classified by the amount of commodity capacity and propulsive capability it possesses. Elements can be divided into two classes: non-propulsive elements M_N and propulsive elements M_P. The element parameters are (Figure 1) as follows:

- The maximum fuel mass of a propulsive element m, $m \in M_P$ is denoted by $m_f{}^m$.
- The specific impulse of the fuel in element m is denoted by $I_{sp}{}^m$.
- The structural mass of element m is denoted by ms^m.
- The mass capacity of element m is denoted by CM^m.
- The volume capacity of element m is denoted by CV^m.
- The cost of element m is denoted by $Cost^m$.

Networks

In order to transfer the commodities and elements from the origin node to the destination node, the trajectories must be defined. The purpose of the interplanetary logistics model developed in this article is to analyze the multiple choices available for routing all of the commodities and elements to determine the best logistics architecture. To model the different available trajectories, a network model of space is created to represent the possibilities available for transferring commodities to their respective destination. The following sections detail the development of the space network utilized to form the model presented in this article.

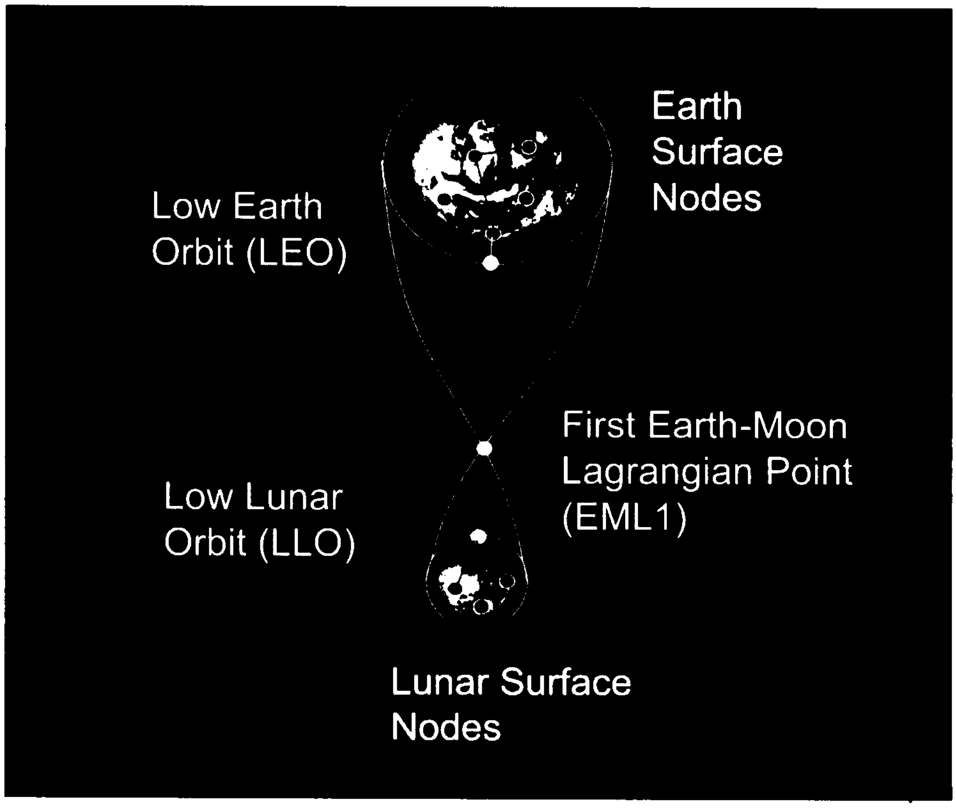

Figure 2
Depiction of an Earth-Moon Static Network

Static Network

The physical network, or static network, represents the set of physical locations, or nodes, and the connections, or arcs, between them. The physical nodes, or static nodes, represent the different physical destinations in space, including the origin and destination of all the commodities, as well as the possible locations for transshipment. Three types of nodes have been identified: Body nodes, Orbit nodes and Lagrange point nodes. These classifications distinguish the type of information required to define a node of each type. The physical arcs, or static arcs, represent the physical connections between two nodes, that is, an element can physically traverse between these two nodes. We define an arc (si, sj) to be a static arc that represents a feasible transfer from static node si to static node sj.

The mathematical description of the static network is given below:

- Define the static network as a graph GS, where $GS = (NS, AS)$.
- Define the set of nodes, $NS = \{s1,..., sn\}$, in the static network.
- Define the set of arcs, $AS \leq NS \times NS$ in the static network.

An example of an Earth-Moon static network is provided in Figure 2. In this picture we can see the connection of the Earth surface nodes to the Earth orbit node, representing launches and returns. Similarly, the lunar surface nodes are connected to the lunar orbit node, representing descent and ascent trajectories. In addition, the orbit nodes, as well as the first Earth-Moon Lagrangian point, are connected by in-space trajectories.

Time Expanded Network

In order to analyze sequences of missions that evolve over an extended period of time, and to account for the time-varying properties that can arise in certain astrodynamic relationships, we have chosen to introduce time expanded networks as a modeling tool. In the time expanded network the absolute time interval under consideration is discretized into T time periods of length Δt. A copy of each static node is made for each of the time points and the nodes are connected by arcs according to the following rules:

- The arc must exist in the static network.
- The arc must create a connection that moves forward in time.
- The arc must represent a feasible transfer, with respect to the orbital dynamics.

The mathematical description of the time expanded network is given below:

- Define the time expanded network as a graph G, where $G = (N, A)$.
- Define the set of nodes in the time expanded network as $N = \{i = (si, t) \mid si \in NS, t = 1,..., T\}$. To simplify the notation, for a given node $i \in N$, let $s(i)$ and $t(i)$ denote the physical node and the time period corresponding to node i, i.e., if

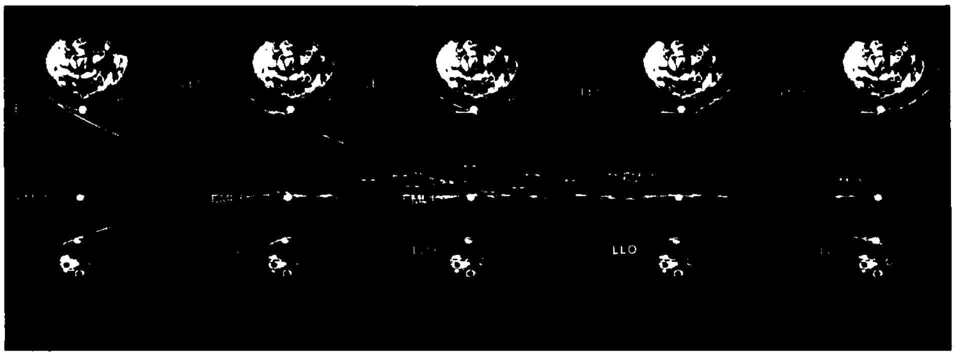

Figure 3
Depiction of an Earth-Moon Time Expanded Network

$i = (si, t)$ then $s(i) = si$ and $t(i) = t$.

- Define node s as the general source that generates the supply of elements. This node is connected to every node in the network where an element can originate.
- Define the set of arcs in the time expanded network as $A \leq N \times N$. An arc $a = (i, j) = ((si, t), (sj, t + T'_{si,sj}))$ exists if and only if there exists an arc (si, sj) in the static network, and the transit time from static node si to static node sj starting at time t is $T'_{si,sj}$. Note that if $si = sj$, then $T'_{si,sj} = 1$ for all t.
- Define path p as a sequence of nodes. In particular, let $f(p)$ and $l(p)$ denote the first node and the last node of path p. If path p originates at node s, $f(p) = s$ for all such p.

Using the static network depicted in Figure 2, we can create the time expanded network in Figure 3. Here, the time expanded network is notional as not all arcs are represented, but how the trajectories evolve in time can be readily seen.

To account for the fact that on certain transfer arcs two burns occur, we slightly modify the time expanded network. We first introduce a new fictitious static node labeled *fic*. Note that this node is not related to the static network. On every transfer arc (i, j), $s(i) \neq s(j)$ requiring two burns we add a new auxiliary node $k = (fic, t)$ with two arcs; one connects i to k and the other one k to j. The value of t is irrelevant. In this new network, each arc (i, j) with $s(i) \neq s(j)$ corresponds to a single burn. All such arcs are called burn arcs and we denote the set of all burn arcs as A_B.

The fuel mass fraction, which represents the ratio of the fuel mass to the initial mass, for element m to execute the burn corresponding to arc $a \in A_B$ is defined as:

$$\phi_a^m = 1 - \exp\left(\frac{-\Delta V_a}{I_{sp}^m g_0}\right).$$

which is taken from the rocket equation.[1]

II. Problem Decomposition

The execution of a space mission requires logistical decisions at every step. Logistics are required to accumulate all of the required commodities for space missions, as well as procure and assemble all elements at the launch site. However, since at the time of launch all of the items required to perform a space mission are co-located at the launch pad, the terrestrial logistics can be decoupled from the interplanetary logistics model. Therefore, the interplanetary logistics model encompasses all of the logistical decisions required between the launch pad and the locations in-space.

There are numerous decisions made during space missions that can be modeled and optimized to create a better mission description. Although, from a system perspective, it would be desirable to make all of these decisions concurrently, due to computational limitations this is not a reasonable approach. Instead, the interplanetary logistics model is decomposed into three fundamental components: launch packing and scheduling, element packing and in-space network optimization.

Launch is a highly constrained transportation activity, where although traditional allocation and packing decisions are required, many additional constraints are necessary to model a feasible launch. For this reason the launch problem is decoupled at Low Earth Orbit (LEO), creating a boundary between the launch allocation and the in-space network optimization. This assumption is assumed to be only slightly restrictive, since for many mission architectures there exists a delay at LEO before proceeding to in-space destinations. Launching focuses on selecting the appropriate elements to perform the launch, satisfying the payload requirements for launch, and scheduling requirements for launch vehicles and launch sites.

Element packing is performed once all of the commodities and element routes have been determined. Given the assignment of commodities and elements to routes optimized in the in-space network optimization, commodities are assigned to elements. In this section, constraints focus on feasible assignments while minimizing transfers.

In-space network optimization examines the entire mission design space of routing from LEO to all locations in-space. Due to the size of the time expanded network that is generated this problem can become quite large, with millions of variables and thousands of constraints. The decision space of the in-space network optimization focuses on the routing of both commodities and elements to routes, and the assignment of elements to burns. The remainder of this article focuses on the formulation of both the variables and constraints required to define the in-space network model.

III. Formulation

Having defined the network, commodities and elements, the in-space network model is presented next. The model is developed in three stages. First, the flow of commodities is defined and the constraints governing the commodity flows are presented. Next, the element flows are modeled with the corresponding constraints. Finally, the constraints governing the capacity and capability, which represent the coupling constraints between the commodities and elements, are developed.

Assumptions

In order to define the mathematical model for the in-space network optimization, the modeling assumptions are first presented. The following assumptions about the behavior of elements are made to create a computationally tractable model.

Consecutive Burns: When an element performs a burn it is defined as an active element. An active element burns only on consecutive burns. Once an element becomes active it stays active for a certain number of burns. As soon as it becomes passive it can no longer be active. Between two consecutive burns, an active element can be idle for an arbitrary length of time by traveling on waiting arcs in the time expanded network. The number of consecutive burns is not constrained.

Fuel Consumption: We assume that before every initial burn the active element is filled to capacity with fuel, and after the burns are completed the remaining fuel is expelled.

Docking/Undocking: We assume that any two elements can be docked and undocked. In addition, if any cost is associated with these operations, it is not explicitly captured. If some elements cannot be docked together, then this must be captured in a post optimization analysis.

The first two assumptions eliminate the need to track the consumption of fuel by each element allocated within the network. Enforcing the final assumption eliminates the requirement of tracking the position of each element in the stack, as the stack can continually reconfigured.

Commodity Flows

Commodity Path Feasibility

In order to understand how each commodity moves through the network it is necessary to determine the path followed from the origin node to the destination node where the commodity fulfills the specified demand. If we define a path variable p, then for each commodity k it is possible to determine a set of feasible paths P^k. For a given commodity k, the path p is feasible only if it originates at node $i = (so^k, t)$ with $t \in to^k$, terminates at node $j = (sd^k, t')$ with $t' \in td^k$, and contains the nodes $w = (sw^k_l, ts^k_l)$ through $w = (sw^k_l, te^k_l)$ where $ts^k_l \in tw^k_l$, $te^k_l \in tw^k_l$ and $te^k_l - ts^k_l = pw^k_l$ for every l, $0 \leq l \leq nw^k$.

Commodity Flow Variables and Constraints

We need to determine how many units of commodity k are transported on path p, for any k and $p \in P^k$. Therefore, for every k and $p \in P^k$ we have a decision variable $x^k_p \geq 0$ such that:

x^k_p = # of units of commodity k on path p.

In order to satisfy the demand d^k of a given commodity x^k_p, we have:

$$\sum_{p \in P^k} x^k_p = d^k$$

for every commodity k.

Element Flows

Element Flow Variables

As defined in Section II, elements can be classified as non-propulsive or propulsive elements, based on whether the element can carry fuel. This distinction allows for two sets of variables to be defined for elements. For any non-propulsive element $m \in M_N$, let us define the decision variable y^m_p such that:

$$y^m_p = \begin{cases} 1 & \text{if non-propulsive element } m \text{ travels on path } p \\ 0 & \text{otherwise,} \end{cases}$$

otherwise, for each feasible path p in the time expanded network. For any propulsive element $m \in M_P$, let us define $z^m_{p,q}$ as the decision variable such that:

$$z^m_{p,q} = \begin{cases} 1 & \text{if element } m \text{ travels on path } p \text{ and is active during sub-path } q \text{ of path } p \\ 0 & \text{otherwise,} \end{cases}$$

where p is any feasible path in the time expanded network and q is a sub-path of p. Note that $\Sigma_q Z^m_{p,q} = 1$ if and only if element $m \in M_P$ travels on path p.

For each path p, the element m can be active on at most one sub-path q. Note that some arc $a \notin A_B$ may be included in the active sub-path q, since an element can be active on a burn arc and then traverse waiting arcs before being active on a consecutive burn arc. Finally, it is possible for a propulsive element to be utilized as a non-propulsive element. For this situation, q is empty.

Element Flow Constraints

The element flow constraints govern the feasibility of element selections. The following constraints govern both propulsive and non-propulsive elements as indicated:

$$\sum_{k}\sum_{p:a\in p} m^k x_p^k \leq \sum_{m\in\mathcal{M}_P}\sum_{p:a\in p}\sum_{q} CM^m z_{p,q}^m + \sum_{m\in\mathcal{M}_N}\sum_{p:a\in p} CM^m y_p^m$$

$$\sum_{k}\sum_{p:a\in p} v^k x_p^k \leq \sum_{m\in\mathcal{M}_P}\sum_{p:a\in p}\sum_{q} CV^m z_{p,q}^m + \sum_{m\in\mathcal{M}_N}\sum_{p:a\in p} CV^m y_p^m$$

A non-propulsive element can only travel on a single path,

$$\sum_{p} y_p^m \leq 1 \qquad m \in \mathcal{M}_N.$$

For active elements, we constrain at most one element to be active on any burn arc,

$$\sum_{m\in\mathcal{M}_P}\sum_{p}\sum_{q:a\in q} z_{p,q}^m \leq 1 \qquad a \in \mathcal{A}_B.$$

A non-propulsive element $m \in M_N$ can travel on an arc a only if there is an active element on that arc,

$$\sum_{p:a\in p} y_p^m \leq \sum_{m'\in\mathcal{M}_P}\sum_{p}\sum_{q:a\in q} z_{p,q}^{m'} \qquad a \in \mathcal{A}_B, m \in \mathcal{M}_N.$$

A propulsive element $m \in M_P$ can travel on an arc a only if there is an active element on that arc,

$$\sum_{p:a\in p}\sum_{q} z_{p,q}^m \leq \sum_{m'\in\mathcal{M}_P}\sum_{p}\sum_{q:a\in q} z_{p,q}^{m'} \qquad a \in \mathcal{A}_B, m \in \mathcal{M}_P.$$

Capacity

For space travel, it is necessary that all commodities be transferred by elements. As such, we must relate the amount of commodities (both mass and volume) present on an arc to the total capacity available on the arc. The total mass capacity of an arc is defined as the sum over all elements on the arc multiplied by their respective mass capacities. Since propulsive and non-propulsive elements are defined differently, it is necessary to account for elements of each type separately. The total commodity mass on an arc is simply the sum over all commodities on the arc multiplied by the commodity mass. Similar constraints are required to ensure that the volume capacity is satisfied as well. The equations below define the mass and volume capacity constraints, respectively:

(See formula at top of page)

Capability

The capability constraints determine if a given element has enough fuel to perform a burn, given the total mass on a burn arc. Here, the constraint requires that the total fuel of the active element performing the burn on a sub-path q must be enough to carry the total cumulative mass along every arc in q. Let q be an arbitrary sequence of possible consecutive burns and let $a^l = (i^l, j^l)$ be the lth burn arc in q for $l = 1,..., |q|$. Here $|q|$ denotes the number of arcs in q. Let $r(p, q)$ denote the sub-path along path p from the first node of p to the first node of q, if q is not empty.

The resulting constraint family reads:

(See formula at bottom of page)

where

$$\Phi_{q,l}^m = \phi_{a^l}^m \prod_{l'=l+1}^{|q|} (1 - \phi_{a^{l'}}^m)$$

The Complete Model

Since the cost to route commodities is negligible, we include only the cost associated with elements. The objective function reads:

$$\min \sum_{m\in\mathcal{M}_p} c^m \sum_{\substack{p \\ f(p)=s}} \sum_{q} z_{p,g}^m + \sum_{m\in\mathcal{M}_n} c^m \sum_{p} y_p^m$$

where c^m is the cost of using element m. The model includes all of the

$$mf^m \sum_{p} z_{p,q}^m + M\left(1 - \sum_{p} z_{p,q}^m\right) \geq \sum_{l=1}^{|q|} \Phi_{q,l}^m \times \left[\sum_{m'\in\mathcal{M}_P}\sum_{p:a^l\in p}\sum_{q'} ms^{m'} z_{p,q'}^{m'} + \sum_{m'\in\mathcal{M}_N}\sum_{p:a^l\in p} ms^{m'} y_p^{m'} + mf^m + \sum_{\substack{m'\in\mathcal{M}_P \\ m'\neq m}}\sum_{p}\sum_{q':a^l\in r(p,q')} mf^{m'} z_{p,q'}^{m'} + \sum_{k}\sum_{p:a^l\in p} m^k x_p^k\right]$$

$$m \in \mathcal{M}_P, \text{ path } q.$$

above constraints. In addition, all x variables are non-negative and all z and y variables are binary.

IV. Solution Methodology

The model presented in the previous section is complex and requires the implementation of a sophisticated algorithm in order to obtain good solutions. Due to the number of variables and constraints, and the complexity of the model, heuristic optimization methods are employed. Although heuristic optimization methods are not guaranteed to return optimal solutions, they often return good solutions quickly.

By understanding the structure of the problem and the potential solutions, the heuristic optimization algorithms can be tailored to the specific problem to enhance computational efficiency and quality of solutions. For the in-space network optimization a series of heuristic optimization algorithms are employed to determine a complete solution to the routing and allocation problem. In this section an overview of the heuristic optimization approach is presented, followed by a more detailed description of each component.

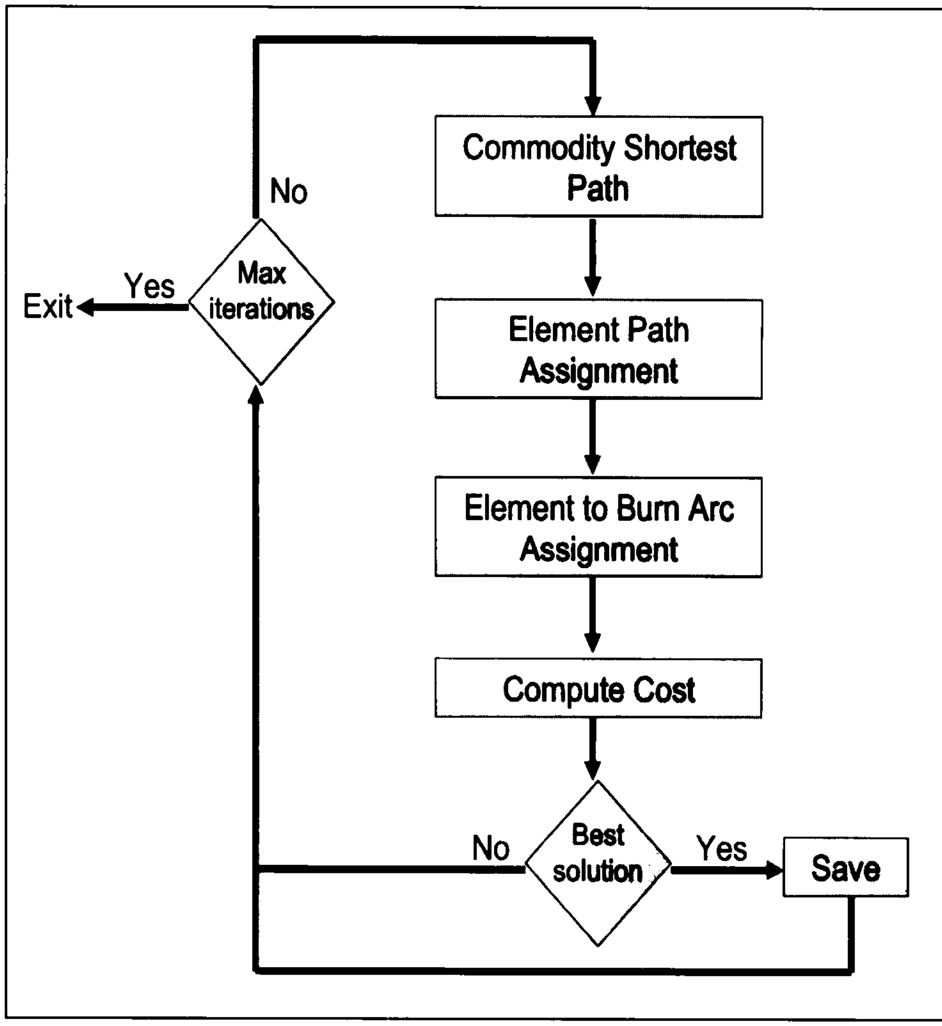

Figure 4
Diagram of Heuristic Optimization

Heuristic Optimization Overview

The optimization of the in-space network has three components: commodity routing, element routing and burn-arc assignment. The commodity routing is performed first, since the entire architecture is driven by the commodity demand. Next, given the commodity paths through the network, elements are assigned to paths, such that all capacity constraints are satisfied. Finally, since the mass of the elements and commodities are known for each arc in the network, the propulsive element assignment can be performed. At several points within the algorithm, randomization is utilized to generate different outcomes and, therefore, this procedure is iterated many times to evaluate the different outcomes. Figure 4 shows the flow of the optimization algorithm.

For each iteration the heuristic optimization determines a feasible set of commodity paths, element paths and burn-arc assignments, sequentially. If a feasible architecture is defined, the cost of the architecture is computed. This cost is evaluated against the cost of the best architecture obtained thus far in the optimization process. If a better architecture is obtained on the current iteration, it replaces the previous best architecture, otherwise, it is discarded. This process is performed until the maximum number of iterations is reached. The remainder of this section provides a detailed explanation of the three components of the heuristic optimization performed for each iteration.

Commodity Routing

Commodity routing is performed by implementing a shortest path algorithm that proceeds as follows. A commodity is selected at random and an auxiliary network is constructed for each path. The auxiliary network connects a single source node to the nodes where a feasible path can begin and a sink node is connected to the nodes where a feasible path can terminate. For commodities that do not have a specified waiting segment, a single auxiliary network is defined where a source node connects the nodes defined by the availability interval, and a sink node connects the nodes defined by the delivery interval.

Given a commodity with *nw* specified waiting segments, *nw* + 1 auxiliary networks are formed. The path is defined backward in time by examining the feasible paths between the nw^{th} waiting segment and the delivery interval. The first auxiliary network created to define this path segment connects a sink node (sk^{nw}) to the delivery interval. The source node (sc^{nw}) is connected to the nodes in the nw^{th} waiting segment defined by (sw_{nw}, t), where $etw_{nw} - pw_{nw} \leq t \leq etw_{nw}$. This definition ensures that the path segment defined will be feasible with respect to the required waiting time period of the specified waiting segment. For each subsequent auxiliary network defined, the sink node connects to the first node in the previously defined path segment and the source node is defined as above. In the final auxiliary network the source node (sc^{1}) is connected to all nodes in the availability interval. Figure 5 depicts a simple example to clarify this method.

For each auxiliary network defined a cost is assigned to every arc. For the first commodity selected the arc costs represent the ΔV of the arcs. Since decreased ΔV correlates to decreased fuel requirements, a shortest path algorithm is implemented to connect the source node to the sink node at lowest cost, or lowest accumulated ΔV. For the remaining commodities the arc costs are defined as $\Delta V (1 - df)^{aN}$ where *df* is a specified fraction and *aN* is the number of times the particular arc has been chosen as an arc in another commodity path. The reduction in cost for previously selected arcs reflects the desire to route commodities on similar paths, where possible.

The shortest path algorithm is run for each auxiliary network of a given commodity until a feasible path is formed between nodes in the availability interval and in the destination interval. This process is then repeated for every commodity until all commodities have been assigned to paths.

Element to Path Assignment

After the commodity paths are determined, the element to path assignment is performed for commodity carrying elements. However, in order to perform this assignment, some preliminary manipulations are necessary. Since the network has arcs that only proceed forward in time, the nodes, and therefore arcs, can be arranged based on this order. This order is known as the topological order, and the details can be found in many network modeling books.[2] A topological order of the nodes and arcs is necessary to ensure that all assignments on downstream connected arcs are determined prior to the current arc assignment.

For each arc in the topological order the following procedure is conducted to ensure that the elements assigned to the arcs for carrying commodities satisfy the mass and volume requirements on each arc. Given an arc in the topological ordering, the total mass and volume of all commodities on that arc is readily computed. To select an element or elements to contain these commodities we first examine forward connecting arcs to determine if a previously assigned element can be reused to contain commodities on the current arc. This process is repeated until both the mass and volume capacity constraints are satisfied or until no existing elements can be utilized.

If additional capacity is required, a new element is selected by utilizing a generalized random adaptive search procedure (GRASP). This algorithm utilizes information about the problem structure and intuition

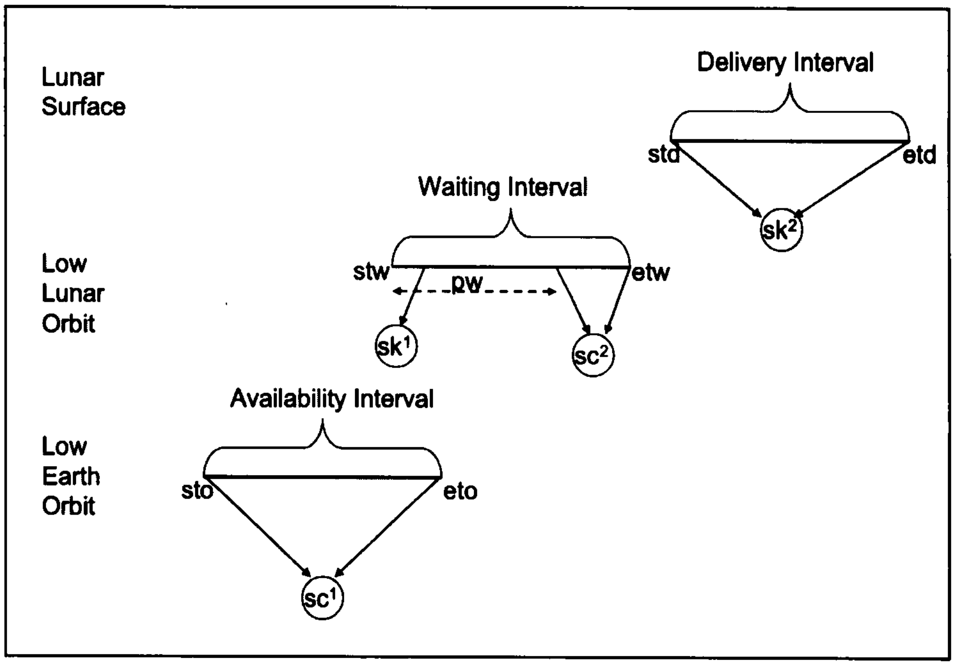

Figure 5
Auxiliary Network Definition for a Commodity with a Specified Waiting Segment

about the characteristics of 'good' solutions to aid in the selection of commodity carrying elements. The algorithm proceeds as follows. One of the six score functions shown in the equation below is selected uniformly at random, and each element is evaluated against the selected score function. The probability of selecting a given element is defined as the negative exponent of a given element's score (i.e., $exp - S_i$) divided by the accumulated probability of every element. This probability distribution favors elements of low cost and high mass capacity. To select a given element, a random number is generated and evaluated against the probability distribution defined above. An element is selected if the random number that has been generated falls into the region in the distribution corresponding to that element. The process of element selection is repeated until all the mass and volume requirements are satisfied for a given arc. The element assignment process continues by working in backward topological order until the mass and volume capacity constraints are satisfied on every arc. From this information, paths for each of the elements can be constructed.

$$S_1 = \frac{Cost}{CM} \quad S_2 = \frac{Cost^2}{CM}$$
$$S_3 = Cost \quad S_4 = \frac{\sqrt{Cost}}{CM}$$
$$S_5 = \frac{Cost}{CM^2} \quad S_6 = \frac{Cost}{\sqrt{CM}}$$

Class of Supply	Demand	Starting Node	Starting Time Interval	Ending Node	Ending Time Interval	Mass	Volume
Equip.	42	LEO	1, 17	Apollo 17	11, 17	10 kg	.5 m^3
Crew	2	LEO	1, 17	Pacific	11, 17	100 kg	2 m^3
Crew	1	LEO	1, 17	Pacific	11, 17	100 kg	2 m^3

Table 1
List of Commodities and Properties for Apollo 17 Example

Class of Supply	# Wait Arcs	Wait Node	Wait Period	Wait Interval
Exploration	0			
Crew	1	Apollo 17	3	7,13
Crew	1	Lunar Orbit	5	7,13

Table 2
List of Commodities and Properties for Apollo 17 Example Continued

Element Type	Fuel Mass	Isp (sec)	Structural Mass	Mass Capacity	Volume Capacity	Number Available	Cost (mil)
Saturn V 1st Stage	2150999	304	135218	0	0	4	692
Saturn V 2nd Stage	451730	421	39048	0	0	4	307
Saturn V 3rd Stage	106600	421	13300	0	0	4	151
SLA	0	0	1837	0	0	4	0.9
Command Module	0	0	5806	100	1	4	148
Service Module	18413	314	6110	0	0	4	118
LM Descent Stage	8156	311	1984	500	5	4	57
LM Ascent Stage	2358	311	2189	100	1	4	79

Table 3
List of Elements and Properties for Apollo 17 Example

Element to Burn Arc Assignment

The final stage of the heuristic optimization is to assign elements to burn-arcs. An element can be assigned to perform a burn if the amount of fuel available in an element is enough to satisfy the capability constraints, and can therefore provide the required ΔV, given the total mass on the arc, as defined by the rocket equation.[1] Since both the commodity paths and non-propulsive element paths are known, the total mass on every arc is known.

Given an arc in the topological order, an element to burn arc assignment is performed as follows. First, forward connecting arcs are examined to determine if a previously allocated propulsive element that has already been utilized to perform a burn on a connecting burn arc can perform an additional burn. If an assignment has not been made, then a check of all elements on the current arc is performed to determine if a commodity carrying element could perform the burn. This second situation is distinguished from the first situation because an element that has propulsive capabilities but is assigned to carry commodities is not automatically assumed to be fueled. Thus, selecting a commodity-carrying element to perform the burn requires additional mass be added to the current arc and all previous arcs in the element's path, to account for the fuel of this element.

If the assignment has not yet been made for the given arc, a new element must be added to the architecture to perform the burn. A new element is selected by employing a GRASP optimization approach, as described above, using the six score functions provided in the above equation, with the replacement of fuel mass capacity *(mf)* for commodity capacity *(CM)*. This situation repeats until a selected element satisfies the capability constraint for the given burn-arc. Since this element is new to the architecture, it is necessary to immediately define the path of the propulsive element and update the payload mass on every arc in the path up to this current burn-arc.

V. Apollo 17 Example

For such a complex problem it is helpful for understanding the model to examine a well defined problem. Using the Apollo 17 elements, a simple example has been defined to determine how the variables above would be defined. The example has three commodities that need to be sent to the Apollo 17 landing site. The commodity properties are listed in Table 1 and 2.

In addition to the commodity properties, the properties of the elements available to both contain and transport the commodities must be defined. A list of these elements is provided in Table 3.

Figure 6 depicts the solution for this example. As we can see, all three commodities are shipped together from LEO. Upon arrival at lunar orbit the commodity associated with the two crew members travels directly to the surface, where it remains for three days. The remaining two commodities wait in lunar orbit until the exploration equipment can be delivered on day 11. The remaining crew member waits in lunar orbit to rejoin the other two crew members before returning to Earth.

Notice that in Figure 6 the crew travel in a lunar module (LM) descent stage. This is a feasible solution since the LM has enough capacity to hold the two crew members. Considerations, such as a feasible element assignment for a given commodity are not handled within the in-space network optimization framework and are addressed in the commodity to element assignment. In situations such as this, a post-optimization analysis would be required to substitute a suitable element for crew transport, if such constraints were desired.

This solution is not only feasible, but represents a good architecture, given the constraints on the commodity paths. If the delivery interval was expanded for the exploration equipment, the optimizer could then choose to combine both the exploration equipment and the two crew members for the surface descent. However, increasing the waiting intervals for the two commodities corresponding to the crew would not affect the optimal solution, since it is desirable for the two commodities to travel together on the return trip to Earth.

VI. Conclusion

In order for space exploration to be sustainable, interplanetary logistics must be considered during mission planning. Research conducted in the terrestrial logistics and operations research communities provides a wealth of modeling tools and solution approaches that can be extended to enable interplanetary logistics decisions. This article explores the requirements necessary to define the interplanetary logistics problem and extends a modeling tool traditionally utilized in terrestrial logistics to incorporate the astrodynamic relationships of space travel.

Using the time expanded network as a decision framework, a complex mathematical model was developed to incorporate the fundamental constraints of in-space transportation. Due to modeling complexities and problem size, a heuristic optimization algorithm was developed to explore the design space and find good solutions to the complex problem. This

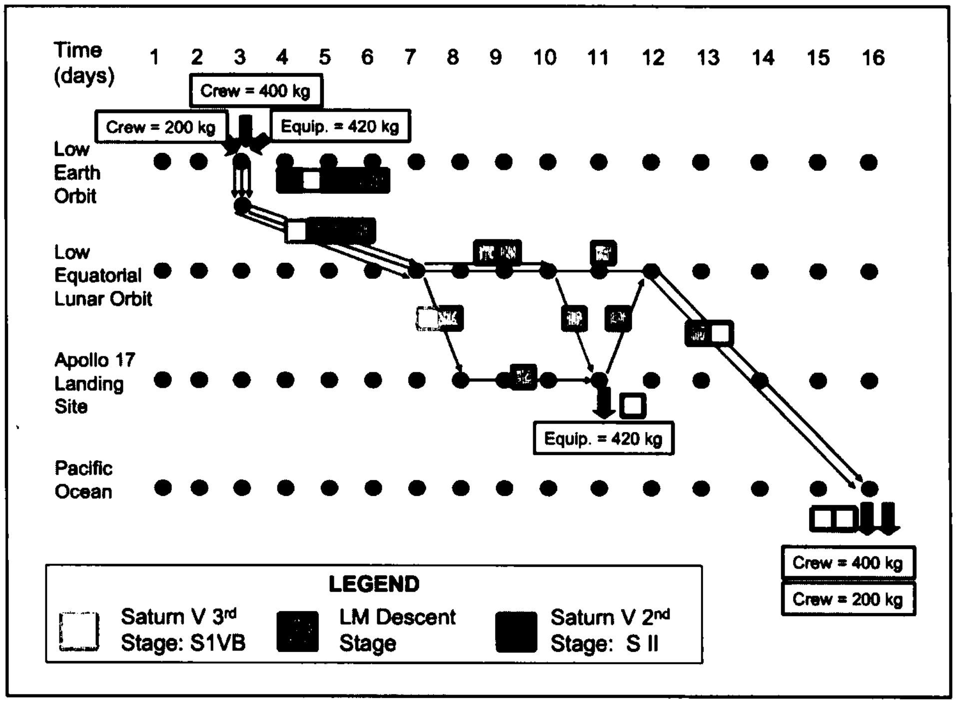

Figure 6
Apollo 17 Example

methodology was demonstrated for the example of an Apollo-style mission to both clarify and validate the model.

Continuing work on this methodology includes incorporating more fidelity into the model to more accurately capture the requirements of space travel. Specifically, the incorporation of gain and loss factors for commodities captures the dependence of the amount of a commodity on the shipment path. Including gain and loss factors creates a trade-off between pre-positioning of commodities and the extra commodity mass required to satisfy the specified demand. Improvements in the solution approach can also be obtained by utilizing the optimization methodology presented as an initial solution to a more robust optimizer, such as CPLEX. Finally, the design space can be expanded to include low-thrust propulsion elements, which in turn requires the definition of corresponding pathways in the network.

End Notes

1. Battin, Richard H., ***An Introduction to the Mathematics and Methods of Astrodynamics***, Revised Edition, AIAA Education Series, 1999.

2. Ahuja, Ravindra K., Magnanti, Thomas L. and Orlin, James B., ***Network Flows: Theory, Algorithms and Applications***, Prentice Hall, 1993.

Author's Biography

Dr. Christine Taylor *received her doctoral degree in February of 2007 from Massachusetts Institute of Technology (MIT) in space systems optimization. Her research focused on incorporating operations planning into the design of aerospace vehicles and adapting classical optimization techniques for complex system of systems problems. She has a bachelor's degree from Cornell University in mechanical and aerospace engineering and a master's degree from MIT in aero/astro. During her studies she was a co-op at Pratt and Whitney, working as both a Vibrations Test Engineer and a Design Engineer. Dr. Taylor was the winner of the Zonta International Amelia Earhart Fellowship in 2005 and was a Draper Laboratory Fellow from 2001 to 2003. Dr. Taylor has co-authored many papers for American Institute of Aeronautics and Astronautics conferences.*

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Logistics Spectrum Publication Schedule

In order to make the *Spectrum* a true international journal, we are looking for more articles from around the world. We would like to hear from our SOLEurope and Asian/Pacific friends. If you've considered writing an article for the *Spectrum*, now is the time to do it! Below is the *Spectrum* publication schedule through the 2nd quarter of 2008. If you would like to write an article that fits into one of the following themes, please contact the appropriate Theme Coordinator. We will, however, accept articles that do not fit into one of the scheduled themes. For all general articles, please send them to *maya.detzel@ae.ge.com.*

Publication Date	Deadline for Articles	Theme	Theme Coordinator	e-Mail address
2nd quarter 2007 (June 1)	April 1, 2007	Humanitarian & Disaster Relief Logistics	Dr. Joanne Stone Wyman	wymanj@battelle.org
3rd quarter 2007 (September 1)	July 1, 2007	Logistics Transformation/Sense & Respond Logistics	Louis A. Kratz	louis.kratz@lmco.com
4th quarter 2007 (December 1)	October 1, 2007	Performance Based Logistics	Anthony E. Trovato, CPL	atrovato@socal.rr.com
1st quarter 2008 (March 1)	January 1, 2008	Logistics Standards	Peter R. Benson	peter.benson@eccma.org
2nd quarter 2008 (June 1)	April 1, 2008	Medical Logistics	Dr. Joseph Curtis	curtisj@usa.redcross.org